UNIVERSITY OF AMSTERDAM

AMSTERDAM CENTER FOR LAW & ECONOMICS

DIFFERENTIATION THROUGH LEGAL UNCERTAINTY

Ryan Bubb

Giuseppe Dari-Mattiacci

Amsterdam Law School Legal Studies Research Paper No. 2024-08

Amsterdam Center for Law & Economics Working Paper No. 2024-04

NEW YORK UNIVERSITY SCHOOL OF LAW

LAW AND ECONOMICS RESEARCH PAPER SERIES
WORKING PAPER NO. 24-23

Differentiation Through Legal Uncertainty

Ryan Bubb and Giuseppe Dari-Mattiacci

October 2025

Differentiation through Legal Uncertainty*

Ryan Bubb† and Giuseppe Dari-Mattiacci‡

October 8, 2025

Abstract

This paper challenges the conventional view of legal uncertainty as a purely distortionary force. We argue that for heterogeneous populations, simple legal standards can harness uncertainty to produce socially beneficial differentiation in incentives. We identify and formalize two mechanisms through which this occurs: a *smoothing channel*, where uncertainty breaks up the inefficient bunching of behavior that would otherwise be caused by a known standard, and a *projection channel*, where individuals rationally form differentiated beliefs about what the standard requires based on their own characteristics. We show that, while uncertainty worsens incentives for mid-cost types, it improves them for more extreme types and can raise aggregate welfare depending on the population mix. We apply our analysis to shed new light on a range of fundamental issues in legal design, including the optimal degree of legal complexity, the choice between rules and standards, and the choice between "sanctions" and "prices."

Keywords: differentiation, legal uncertainty, standards, reasonable person, false consensus
JEL codes: D02; D83; K10; K13, K40

*For helpful comments we are grateful to Barry Adler, Oren Bar-Gill, Will Baude, Omri Ben-Shahar, Lee Fennel, William Hubbard, Aziz Huq, Marcel Kahan, Daniel Klerman, Louis Kaplow, Lewis Kornhauser, Daryl Levinson, Murat Mungan, Josh Teitelbaum, David Weisbach, Abe Wickelgren, and seminar participants at Columbia Law School, Goethe University, NYU School of Law, University of Hamburg, University of Chicago Law School, University of Texas at Austin School of Law, and the 2022 Annual Meeting of the American Law and Economics Association at Columbia Law School.

†University of Southern California, rbubb@law.usc.edu.
‡University of Amsterdam, gdarimat@uva.nl.

1 Introduction

A fundamental challenge in legal design is in creating rules that provide appropriately differentiated incentives across heterogeneous individuals and circumstances. This issue is especially pronounced for "sanctions"-based legal strategies, which combine a behavioral norm defining required conduct with a penalty for noncompliance (Cooter, 1984). Sanctions strategies like the negligence rule represent a comparatively centralized mode of legal ordering—the state effectively micromanages individuals by specifying behavioral norms—and consequently require substantial information about socially appropriate behavior in diverse circumstances. In contrast, "pricing" strategies such as strict liability adopt a more hands-off approach, allowing individuals to make their own choices provided they pay for any resulting external costs. Due to their substantial informational demands, sanctions strategies inevitably differentiate less in practice than would an ideal system. For example, the reasonable person standard in tort law is generally understood to be an objective standard, based on the care that would be reasonable for the average person, taking only limited account of variation across individuals.[1]

Another barrier to explicit legal differentiation stems from legal uncertainty—individuals' incomplete information about the precise legal consequences of alternative actions. Such uncertainty is pervasive and arises from multiple sources, including incomplete information available to the law enforcement system and individuals' limited understanding of substantive law. Even if the state's information costs are low, and it can devise a finely tailored set of legal rules, such a regime will not achieve the desired results if individuals do not learn the rules (Kaplow and Shavell, 1996). Moreover, even when the expected legal standard aligns perfectly with socially optimal behavior, uncertainty can still distort legal incentives (Craswell and Calfee, 1986). On one hand, legal uncertainty may motivate over-compliance as individuals seek to reduce their risk of sanctions. On the other hand, high levels of uncertainty can dilute legal incentives, resulting in under-compliance.

[1] Restatement (Second) of Torts § 283; van Dam (2014).

In this paper, we develop a very different perspective on legal uncertainty. While often seen as a source of costly distortion, we show how legal uncertainty can also serve as a valuable lubricant for the legal system. The friction it addresses arises when a simple, one-size-fits-all legal standard is applied to a diverse population. When such a standard is certain, heterogeneous parties get "stuck," inefficiently bunching their behavior at the single required level of conduct. While introducing legal uncertainty distorts the behavior of individuals for whom a uniform legal standard is set at the approximately optimal level, it also improves the behavior of more extreme types by freeing them from rigid adherence to ill-fitting standards.

This lubricating effect of legal uncertainty thus allows simple legal standards to operate more smoothly across heterogeneous populations, harnessing decentralized information to produce differentiated incentives without the costs of explicit legal differentiation. Somewhat paradoxically, this lubricating effect emerges from a collision of two informational gaps. The state's limited information forces reliance on non-differentiated standards, while individuals' limited information allows these rigid standards to nonetheless accommodate heterogeneity. In other words, the incompleteness of individuals' information in navigating the legal order can mitigate the incompleteness of the state's information in designing the legal order.

We identify two distinct channels through which legal uncertainty can produce more differentiated incentives: the *smoothing channel* and the *projection channel*. The smoothing channel operates by eliminating the discontinuities in incentives that coarse behavioral standards would otherwise create. The projection channel functions through individuals forming rational beliefs about what simple standards require based in part on their own circumstances.

We develop this framework by extending the canonical economic model of the reasonable person standard in tort law (Brown, 1973; Shavell, 1987; Landes and Posner, 1987). In the standard setup, potential injurers with varying costs of care face a negligence rule based on a uniform standard of care. This legal rule creates a stark discontinuity in liability costs at the standard of care: at levels of care below the standard, injurers bear the full cost of any accidents they cause; at or above the standard, they face no liability. This discontinuity induces a corresponding discontinuity in injurers' private marginal benefits of care, so in the absence of legal uncertainty, many types bunch at the due-care level.[2]

When potential injurers are uncertain about the liability they face, in contrast, incentives become more differentiated. First, consider the case in which, from the perspective of injurers, the legal system is subject to "legal noise"—random errors in the finding of liability that are orthogonal to the characteristics of any particular case. Consequently, there is no longer a level of care at which injurers' expected marginal liability costs fall discontinuously to zero. Rather, under legal noise, individuals' expected marginal liability costs vary smoothly with their level of care, leading individuals with higher costs of care to choose lower levels of care.

Our formal analysis of the smoothing channel provides a precise, functional-form-free, type-by-type characterization of uncertainty's welfare effects, which reveals a fundamental trade-off. For injurers with costs near the population average, legal uncertainty unambiguously harms welfare by increasing their social costs. This is because the simple legal standard is already set at the optimal level for the average-cost type; under certainty, this injurer takes their first-best level of care. When uncertainty is introduced, the competing incentives it creates—the dilution of deterrence versus the motive to over-comply to avoid any chance of liability—push this injurer away from the social optimum. Since any deviation from this unique first-best level of care is necessarily inefficient, uncertainty acts as a pure distortion for the average type. This result formally nests the classic distortionary effects found in the prior literature (Craswell and Calfee, 1986).

In contrast, our key novel finding is that for more extreme types who would comply with the standard under certainty despite it being poorly suited to their circumstances, le-

[2]In Appendix B we show that our results remain valid in a model with incremental damages that do not increase discontinuously at the due-care level (as in Grady, 1983; Kahan, 1989).

gal uncertainty is affirmatively welfare-enhancing. For these injurers, the certainty benchmark is not first-best care but rather inefficient "bunching" at a uniform standard. The differentiation induced by the smoothing channel frees them from this rigid adherence, resulting in them choosing care levels closer to their social optima, thereby lowering their contribution to total social costs. As a result of these competing effects, whether legal uncertainty ultimately raises or lowers aggregate social costs depends on the population mix—specifically, on the degree of heterogeneity across injurers.

The projection channel complements the smoothing channel by endogenizing beliefs about simple standards. Such beliefs are the proximate cause of law's incentive effects on behavior. When we say that individuals are deterred from engaging in some socially destructive behavior, we mean that they avoid such behavior because they believe they face a less attractive lottery over legal consequences if they engage in it than if they do not. It is the individuals' beliefs about the law that are doing the incentive work, not the law *per se*. If this distinction seems pedantic, that is only because the dominant paradigm in the economic analysis of law assumes that systematic variation in legal beliefs stems solely from variation in the actual legal treatment of cases. However, we identify a mechanism through which, in the absence of explicit legal differentiation, Bayesian individuals form beliefs about what the law requires that correlate with what would ideally be required from each of them.

Consider again the canonical model of the reasonable person standard and suppose potential injurers know they must take reasonable care but do not know what is reasonable. They thus form Bayesian beliefs about due care based on their beliefs about the average cost of care in the population. Suppose injurers have a common prior about the cost distribution and that each has a single draw from the distribution, namely, their own cost of care. As a result, injurers with lower costs of care believe that the average cost of care is lower—and therefore that the level of due care is higher—than do injurers with higher costs of care. This differentiation in beliefs about simple legal standards can produce socially useful differentiation in behavior.

At the heart of the projection mechanism is a sort of parochiality of even rational, Bayesian individuals. Much of what people know about the human condition derives from their own capacities, preferences, experiences, and opportunities. They naturally and rationally form beliefs about others based in important part on their own situation. In a sense, they *overgeneralize* (rationally!) from their own experiences. Moreover, most people interact primarily with similar others—a phenomenon sociologists call *homophily* (Lazarsfeld and Merton, 1954). Individuals' beliefs about others in turn inform their understanding of simple legal standards like "reasonableness." As a result, they will typically believe that the law is closer to what is appropriate for themselves than it actually is.

These two channels through which legal uncertainty produces differentiated incentives—the smoothing channel and the projection channel—can operate together, as in the reasonable person standard, or independently. The smoothing channel functions only when there would otherwise be discontinuities in marginal legal incentives and even when all individuals share identical legal beliefs. The projection channel, conversely, operates only when an individual's own type is informative about the content of the relevant legal standards, which can occur even without discontinuities in marginal legal incentives.

As a result of these mechanisms, legal uncertainty can potentially increase the efficiency of, rather than undermine, the incentives produced by law. These mechanisms thus may help explain the abiding role of simple standards in modern legal systems. The statute books and the Code of Federal Regulations certainly grow longer by the year, detailing ever more precisely the commands of law. One might think that it is inevitable that the broad standards of conduct characteristic of the common law approach will become obsolete and give way in the face of this rising tide of positive law. Yet simple—and vague—standards endure. They remain central to many bodies of law, from the fiduciary duties of corporate officers and directors, to the Sherman Act's prohibition on "contracts, combinations, and conspiracies in restraint of trade," to the obligation each of us is under to act as a reasonable person in like circumstances would to avoid visiting harm on others. Even the Internal Revenue Code, commonly thought to be the paradigmatic system of rules, is replete with standards (Weisbach, 1999).

Our analysis contributes to an existing literature analyzing the optimal degree of differentiation in law. Kaplow (1995) identifies two key considerations in determining how *complex* law should be: the extent of heterogeneity in the regulated activity and information costs. Information costs can result in complex rules failing to achieve the desired differentiation in behavior, for example because individuals do not learn the legal consequences of alternative choices (Kaplow and Shavell, 1996). But as our model shows, legal differentiation is not the only way to produce socially useful variation in incentives. Moreover, information costs are not only a barrier to the legal strategy of explicit differentiation; they also create opportunities for the state to differentiate incentives without incurring the costs of legal differentiation.

The smoothing channel was first introduced in the law-and-economics literature in a debate over the efficiency of comparative negligence rules. Rubinfeld (1987) first identified the basic mechanism in a model where comparative negligence smooths out the discontinuity in injurers' marginal expected liability costs by providing a degree of sharing of liability with victims that varies continuously with injurers' care, arguing that this feature strengthened the case for comparative negligence over standard negligence rules. Bar-Gill and Ben-Shahar (2003) contests this argument by pointing out that the differentiation mechanism that Rubinfeld (1987) attributed to comparative negligence can also arise under other negligence rules. For example, the contributory negligence rule results in victims bearing all accident costs so that victims' incentives are smooth, resulting in differentiated levels of care. They also show that evidentiary uncertainty can smooth out the discontinuity in injurers' incentives under a negligence rule. Tax scholars have similarly recognized how uncertainty about standards governing the classification of an activity for tax purposes can smooth out discontinuities in incentives that would exist under bright-line rules (Weisbach, 1999; Fox and Goldin, 2019). Our analysis of the smoothing mechanism goes beyond this existing literature by providing a more general analysis of how different forms of legal uncertainty can produce social benefits through the smoothing and the projection channels. We clarify the conditions under which the smoothing channel is operative, characterize the resulting welfare effects, and analyze the implications for various issues in legal design, such as the choice between rules and standards.

Our analysis is also related to prior work analyzing potential beneficial effects of legal uncertainty. Lang (2017) develops an adverse selection model of an enforcement authority determining whether firms may lawfully take a specified action and shows that an increase in firms' uncertainty about how they will be evaluated can result in more efficient screening of the firms. Ederer et al. (2018) analyzes how uncertainty about the weights on performance measures in incentive schemes can mitigate the gaming problem. Baker and Raskolnikov (2017) shows that in an environment in which agents seek preapproval for a proposed action from a regulator, an increase in legal uncertainty can raise the welfare of the agent if the regulator is more likely to grant preapproval if the action is deemed to greatly surpass the legal standard. Tabbach and Cohen (2024) show that legal uncertainty does not distort incentives if the parties can "choose" the level of uncertainty by adapting their behavior.

The paper proceeds as follows. In Section 2, we analyze a model of the reasonable person standard and characterize the differentiation in injurers' care levels generated without uncertainty and under different forms of legal uncertainty. In Section 3, we discuss the implications of our analysis for the optimal degree of complexity and personalization of law, the choice between rules and standards, and the choice between sanctions and prices. Section 4 concludes. All proofs are in Appendix A. Appendix B contains an extension of the model to negligence with incremental damages.

2 The Model

Our model builds on the canonical economic model of the reasonable person standard of Shavell (1987). We begin in Section 2.1 by laying out the basic setup and characterizing the socially optimal outcome. Section 2.2 analyzes the model under no uncertainty, briefly recapitulating standard results in the literature. In Section 2.3 we introduce legal uncertainty in the form of "noise" where injurers share common, unbiased beliefs about how courts will evaluate their behavior, allowing us to analyze the smoothing mechanism. Finally, in Section 2.4 we explore a different form of legal uncertainty where injurers do not know the standard of care and form Bayesian beliefs based on their own information and analyze the operation of the projection mechanism.

2.1 Basic setup

Consider a population of potential injurers, each of whom must choose a level of care $x \geq 0$ at private cost cx. Care reduces the expected cost of accidents, $l(x)$, at a diminishing rate:

$$l'(x) < 0, \quad l''(x) > 0$$

with $\lim_{x \to 0} l'(x) = -\infty$, $\lim_{x \to \infty} l'(x) = 0$, and $\lim_{x \to 0} l(x)$ exists. Furthermore, for all $x > 0$, $l'(x)$ remains finite—that is, it never diverges to $-\infty$ on $(0, \infty)$. The cost of care $c \in (0, \infty)$ varies across injurers. We assume that c is distributed according to PDF $f(c)$ with associated CDF $F(c)$.

We take the social objective to be to minimize the total costs of accidents, which include care costs plus the expected residual harms caused by any accidents. This is given by

$$L = \int_0^\infty \left[\left(l(x(c)) + cx(c)\right] f(c)\, dc, \tag{1}$$

where $x(c)$ is the care taken by an injurer of type c. The first-best level of care for an injurer of type c, denoted $x^{FB}(c)$, minimizes the integrand in (1) for each injurer's type and hence is implicitly defined by the first-order condition:

$$l'\left(x^{FB}(c)\right) + c = 0. \tag{2}$$

Because $l''(x) > 0$, the solution is unique and decreases monotonically in c—that is, $\frac{dx^{FB}(c)}{dc} < 0$.

As is well known, a "pricing" regime like strict liability can implement the first-best levels of care across heterogenous injurers (Cooter, 1984). However, we focus on the negligence rule, which is an example of the more general class of "sanctions" legal strategies that specify a standard of behavior and impose a sanction for failure to comply. We do so because sanctions strategies potentially implicate both the smoothing channel and the projection channel. We consider the implications of our analysis for strict liability—and more generally for "pricing" legal strategies—in the discussion in Section 3.

2.2 Reasonable person standard with no uncertainty

Consider a negligence regime in which an injurer who causes an accident must pay damages if and only if their level of care is less than the reasonable person standard of care, s.[3] We assume that courts cannot observe each injurer's private cost of care c but do know the overall distribution of c in the population. The reasonable person standard is taken to be the socially optimal level of care for an injurer whose cost is average.[4] Formally,

$$s = x^{FB}(\bar{c}),$$

[3]We exclude consideration of activity levels and victim care to develop our analysis of differentiation through uncertainty in the simplest setting possible. The same core insights extend to settings that include additional margins of behavior.

[4]The basic mechanisms we identify would also apply if the standard of care were set at some other function of the cost distribution—e.g., the median-cost injurer's first-best level of care—as long as our informational assumptions hold.

where again $x^{FB}(c)$ is the level of care that minimizes $l(x) + cx$ for a cost type c. For now we assume that all injurers *know* the standard of care, s.

Each injurer of type c then chooses x to minimize their *own* total costs of care plus expected liability:

$$\min_{x\geq 0} \begin{cases} cx & \text{if} \quad x \geq s \\ l(x) + cx & \text{if} \quad x < s \end{cases}$$

This problem has a well-known solution: low-cost injurers meet the standard s and avoid liability, and high-cost injurers "opt out" to take their first-best levels of care.

Proposition 1. *Under a known standard of care $s = x^{FB}(\bar{c})$, there is a threshold $\tilde{c} > \bar{c}$ such that each injurer's chosen level of care is*

$$x^*(c) = \begin{cases} s & \textit{if} \quad c \leq \tilde{c} \\ x^{FB}(c) & \textit{if} \quad c > \tilde{c} \end{cases}$$

This result illustrates a key limitation of using a uniform standard of care to regulate heterogeneous injurers: the vast majority of cost types ($c \leq \tilde{c}$) all "bunch" at s. Injurers with $c < \bar{c}$ undershoot their first-best care level because they choose the minimum care necessary to avoid liability. Meanwhile, those in the range $(\bar{c}, \tilde{c}]$ overshoot, choosing s even though less care would be socially optimal.[5] Only the relatively small fraction of types with $c > \tilde{c}$—for whom the costs of complying with the standard outweigh the avoided liability costs and so in effect face a strict liability rule—choose differentiated levels of care.[6] Figure 1 illustrates these outcomes.

[5]Injurers with $c = \tilde{c}$ are indifferent between s and $x^{FB}(c)$, which we arbitrarily resolve in favor of $x^*(\tilde{c}) = s$. In the interest of brevity, we will similarly resolve other ties below without further comment.

[6]For example, under the two assumptions that we make in Section 2.4—c distributed $\mathcal{LN}(\theta, \sigma^2)$ and $l(x) = \frac{1}{x}$—fewer than 5% of injurers have costs higher than the cutoff $\tilde{c}$. In particular, in this case we have $\tilde{c} = 4\bar{c}$ and $F(4\bar{c}) = \Phi\left(\frac{\log[4]}{\sigma} + \frac{\sigma}{2}\right) \geq \Phi\left(\sqrt{2\log[4]}\right) = 0.952$, where Φ is the CDF of the standard normal distribution and $\min_\sigma \left[\frac{\log[4]}{\sigma} + \frac{\sigma}{2}\right] = \sqrt{2\log[4]}$. While this specific quantitative result rests on the lognormality assumption, since the cutoff $\tilde{c}$ is always greater than the average cost of care $\bar{c}$, the percentage of injurers taking first best care will typically be small.

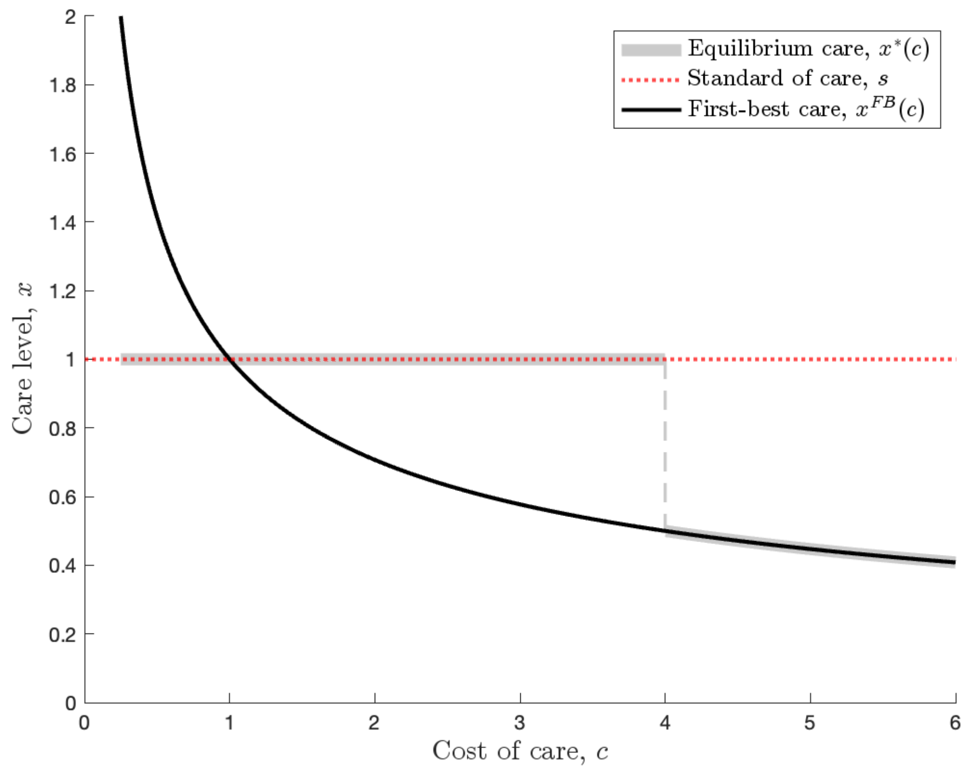


Figure 1: **Care taken with no uncertainty.** Assumptions: $l(x) = \frac{1}{x}$ and $\bar{c} = 1$ so that $s \equiv x^{FB}(\bar{c}) = 1$ and $\tilde{c} = 4$.

2.3 Reasonable person standard with legal noise

Suppose now that injurers are uncertain about how the legal system will evaluate their behavior but share common, unbiased beliefs. Specifically, all injurers view the standard of care s as a random variable with mean $\bar{s}_{\varepsilon} = x^{FB}(\bar{c})$ and variance σ_{ε}^2. We refer to this as the "legal noise" model. Such uncertainty can arise from multiple sources, including imperfect information about how courts will interpret the reasonable person standard and uncertainty in the court's measurement of an injurer's level of care. As argued by Shavell (1987), many different types of legal uncertainty can be modeled isomorphically.[7]

Denote the CDF of injurers' common beliefs about s by $F_{\varepsilon}(\cdot)$, with corresponding PDF

[7]We ignore the possibility that individuals could obtain additional information at a cost in order to reduce legal uncertainty, but all of our positive analysis would go through unchanged if we allowed for that so long as there remained residual legal uncertainty.

$f_\varepsilon(\cdot)$. An injurer of type c chooses their level of care by solving:

$$\min_{x\geq 0}\Big[\big(1-F_\varepsilon(x)\big)l(x)+cx\Big], \tag{3}$$

where $1-F_\varepsilon(x)$ represents the probability that the standard of care exceeds the injurer's care, resulting in liability. The solution to this problem for type c, denoted $x^*_\varepsilon(c)$, must be interior and therefore satisfies the first-order condition:[8]

$$-\Big[1-F_\varepsilon\big(x^*_\varepsilon(c)\big)\Big]\,l'\big(x^*_\varepsilon(c)\big)+f_\varepsilon\big(x^*_\varepsilon(c)\big)l\big(x^*_\varepsilon(c)\big)=c. \tag{4}$$

The left-hand side of (4) represents the private marginal benefit of care, which arises from reduced expected liability. The right-hand side is the marginal cost of care, which varies across injurers. As in Craswell and Calfee (1986), uncertainty about the legal standard has two offsetting effects. First, the term $1-F_\varepsilon\big(x^*_\varepsilon(c)\big)<1$ captures how uncertainty can dilute incentives by reducing the probability of liability. Second, the term $f_\varepsilon\big(x^*_\varepsilon(c)\big)l\big(x^*_\varepsilon(c)\big)$ captures the marginal reduction in the probability of liability from increasing care, which can create incentives for over-compliance.

Unlike the case without uncertainty, where most injurers bunch at the standard of care, legal noise produces differentiated levels of care that vary smoothly with injurers' cost types, as the following proposition states more formally.

Proposition 2. *Under the reasonable person standard with legal noise:*

1. *The chosen level of care $x^*_\varepsilon(c)$ is strictly monotonically decreasing in the injurer's cost of*

[8] Note that the second order condition $(1-F(x))\,l''(x)-2f(x)\,l'(x)-f'(x)\,l(x)>0$ is not necessarily satisfied over its entire domain.Yet, strict convexity is typically, if not necessarily, satisfied. In particular, the first two terms in the second-order condition are unambiguously positive. The sign of the third term, $-f'(x)\,l(x)$, is negative if and only if $f'(x)$ is positive. The objective function is thus globally convex unless, for some $x>0$, $f'(x)\,l(x)$ is sufficiently large to outweigh the first two positive terms. In addition, even if the objective function is not globally convex, in which case the first-order condition in (4) might have multiple solutions, the solution to the injurer's optimization problem is generically unique. This is because the presence of multiple local minima results in multiple global minima only if two or more minima yield exactly the same total cost for the injurer. This occurrences have typically measure zero. Otherwise, the global minimum is unique despite lack of global convexity.

care, c, with $\lim_{c\to 0} x_{\varepsilon}^{*}(c) = \infty$ and $\lim_{c\to\infty} x_{\varepsilon}^{*}(c) = 0$;

2. *Injurers with cost of care $c \leq \frac{\bar{c}}{2}$ over-comply ($x_{\varepsilon}^{*}(c) < s$). For injurers with cost of care $c > \frac{\bar{c}}{2}$ there exists a threshold level of uncertainty $\hat{\sigma}_{\varepsilon}(c)$ decreasing in c such that:*
   - *At low levels of uncertainty ($\sigma_{\varepsilon} < \hat{\sigma}_{\varepsilon}(c)$), the injurer over-complies ($x_{\varepsilon}^{*}(c) > s$)*
   - *At high levels of uncertainty ($\sigma_{\varepsilon} > \hat{\sigma}_{\varepsilon}(c)$), the injurer under-complies ($x_{\varepsilon}^{*}(c) < s$).*

Figure 2 illustrates how legal uncertainty increases differentiation by showing levels of care implemented by the liability system as a function of c under various degrees of legal noise, measured by the standard deviation σ_{ε} of each injurer's beliefs about s. The results reveal several important patterns.

When a small amount of legal noise is introduced, injurers must take somewhat greater care to substantially eliminate liability risk. This causes the cost threshold at which injurers effectively give up on trying to comply with the standard of care to decrease, as shown by the care function with $\sigma_{\varepsilon} = 0.003$ compared to the care function under certainty. Additionally, lower-cost injurers "over-comply" by taking greater care than the expected standard of care.

As uncertainty increases, care becomes increasingly differentiated across the entire domain of c, with the care function approaching an approximation of the first-best care levels. However, at high levels of legal uncertainty (e.g., $\sigma_{\varepsilon} = 3$), legal incentives become significantly diluted, resulting in most injurer types taking less than the standard of due care.

To build intuition about what drives the differentiation in behavior in this model and its welfare implications, Figure 3 plots both the social marginal benefit of care (from the reduction in expected accident costs from greater care, or $-l'(x)$) and the private marginal benefit of care (from the reduction in the injurer's expected liability) under different degrees of legal uncertainty. For a given type c, the first-best level of care occurs where the social marginal benefit of care equals that type's marginal cost of care, c. The figure in-

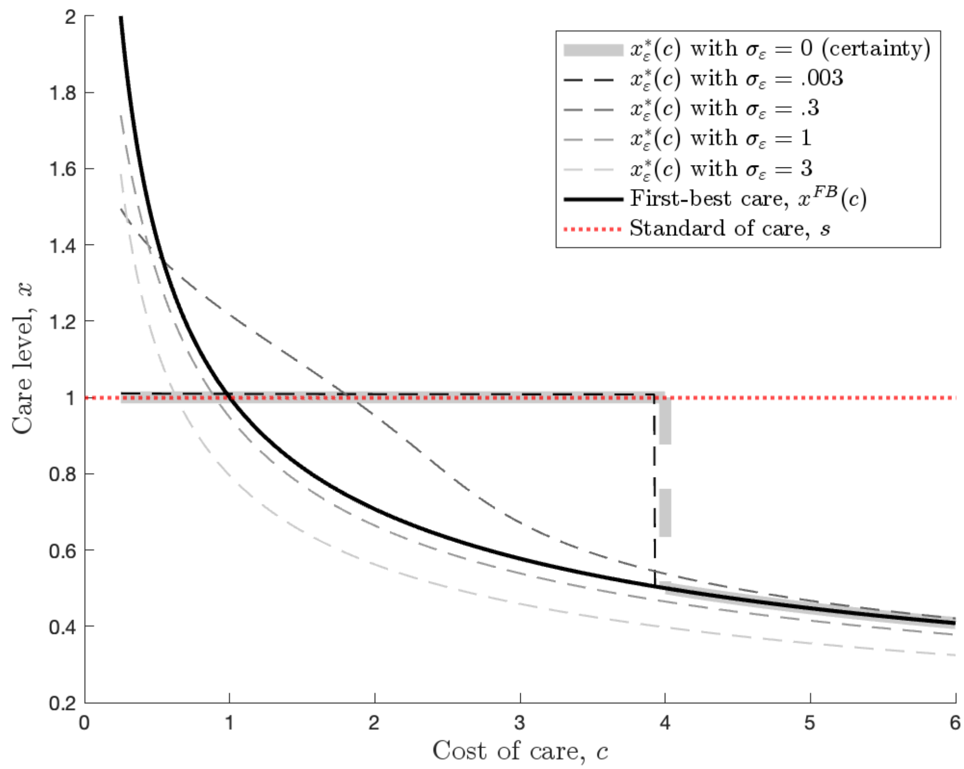


Figure 2: **Care taken with legal noise.** Assumptions: $l(x) = \frac{1}{x}$ and $\bar{c} = 1$ so that $\tilde{c} = 4$ and $s \equiv x^{FB}(\bar{c}) = 1$, with injurers' beliefs about s distributed $\mathcal{N}\left(1, \sigma_\varepsilon^2\right)$ for varying levels of σ_ε.

cludes a horizontal line at one illustrative level of c, denoted c_1, with the corresponding first-best level of care marked at the point where this line intersects the social marginal benefit curve.

Without uncertainty, the private marginal benefit—the thick gray line in the figure—drops discontinuously to zero at the standard of care, above which the injurer bears no liability. This discontinuity leads most injurer types to bunch at the corner solution $x = s$ under certainty.

Under uncertainty, however, the private marginal benefit of care becomes continuous, thereby inducing a unique interior optimum in care choices for each type c. Specifically, with "legal noise" injurers choose care so that their private marginal benefit of care, given by

$$PMB(x) = -\left[1 - F_\varepsilon(x)\right] l'(x) + f_\varepsilon(x) l(x), \tag{5}$$

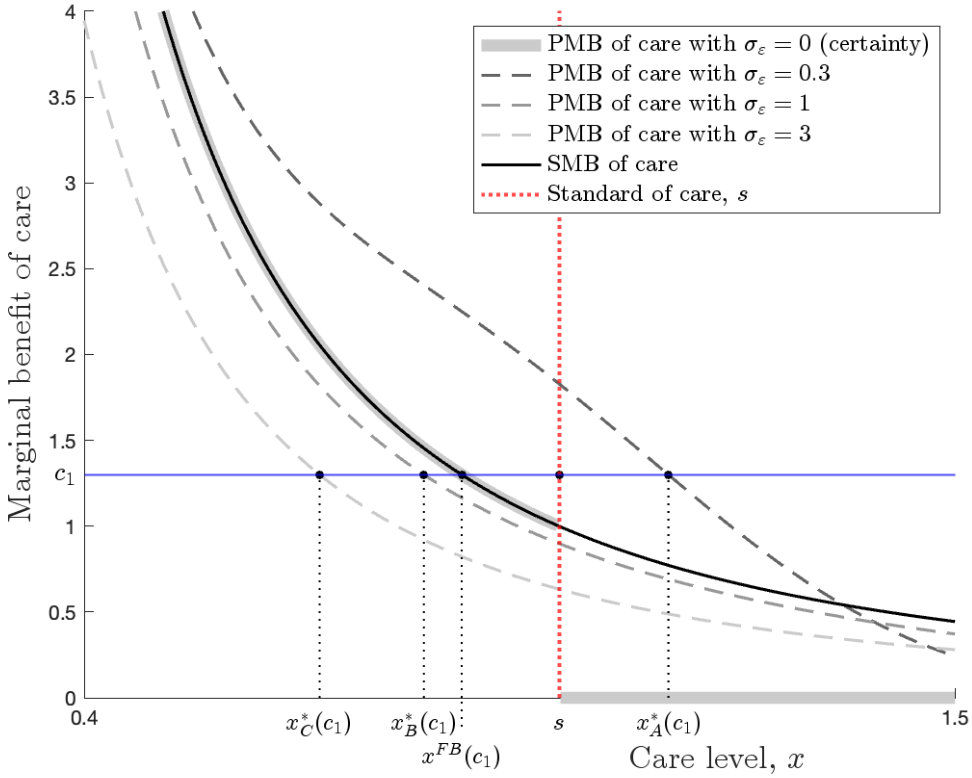


Figure 3: **Marginal private benefit of care with legal noise.** Assumptions: $l(x) = \frac{1}{x}$ and $\bar{c} = 1$ so that $\tilde{c} = 4$ and $s \equiv x^{FB}(\bar{c}) = 1$, with injurers' beliefs about s distributed $\mathcal{N}\left(1, \sigma_\varepsilon^2\right)$ for varying levels of σ_ε.

is equal to their marginal cost of care. This results in care levels that are differentiated according to injurers' costs, c.

Figure 3 shows the private marginal benefit curves for three different levels of uncertainty (measured by σ_ε). A type c's chosen care level occurs where the private marginal benefit equals c. The figure marks these chosen care levels of type c_1 under each of the three levels of uncertainty depicted.

Because the private marginal cost of care c also represents the *social* marginal cost, any type c will choose first-best care if and only if the ratio of private to social marginal benefit at their choice equals 1. This ratio is :

$$\frac{PMB(x)}{SMB(x)} = 1 - F_\varepsilon(x) - f_\varepsilon(x)\frac{l(x)}{l'(x)}. \tag{6}$$

When this ratio evaluated at $x = x_\varepsilon^*(c)$ exceeds 1, type c is over-deterred (that is, takes more care than the first-best level); when it falls below 1, type c is under-deterred. The second term, $-F_\varepsilon(x)$, constitutes what we call the "dilution channel," which tends to push the ratio below 1. The third term, $-f_\varepsilon(x)\frac{l(x)}{l'(x)}$, represents the "over-deterrence channel," which tends to push the ratio above 1. For the average injurer (type $\overline{c}$), the first-best level of care equals the standard of care and hence there is no distinction between compliance and deterrence. For all other injurers, these thresholds are distinct, meaning injurers may over-comply while being under-deterred and vice versa.

Returning to Figure 3, consider the case where $\sigma_\varepsilon = 0.3$. For type c_1, the resulting care level $x_A^*(c_1)$ deviates further to the right of that type's first-best care level than is the standard of care s, which this type would choose under certainty. In this case the over-deterrence effect dominates the dilution effect, resulting in higher total accident costs at this level of uncertainty than under certainty. However, as c increases, the private marginal benefit curve for $\sigma_\varepsilon = 0.3$ eventually crosses s, bringing it closer than s to the social marginal benefit of care. For cost types above this crossing point, legal uncertainty improves care from a social perspective.

With $\sigma_\varepsilon = 1$, the private marginal benefit curve closely approximates the social marginal benefit curve, indicating that the dilution effect and the over-deterrence effect roughly offset each other. For type c_1, the resulting chosen care level lies much closer to the first-best level than does the standard of care, thereby reducing this type's total cost of accidents. But as uncertainty increases further (e.g., $\sigma_\varepsilon = 3$), the dilution effect strengthens, distorting care levels downward, as shown by the corresponding marginal benefit curve.

To characterize these effects more formally, we define the social cost of accidents of type c given level of care x by:

$$T(x;c) = l(x) + cx. \tag{7}$$

Our main welfare result compares social costs of each type under uncertainty, $T(x_\varepsilon^*(c);c)$, with those under certainty. Since injurers with $c \leq \tilde{c}$ comply with the standard while those with $c > \tilde{c}$ opt out (Proposition 1), we proceed in two steps: first comparing all types to compliance with a known standard to isolate the smoothing effect (Proposition 3), then incorporating opt-out behavior for a complete welfare assessment (Corollary 1).

Proposition 3 (Social costs compared to compliance with a known standard)**.** *Under the reasonable person standard with legal noise:*

1. *There exists a threshold $c_1 \leq \bar{c}$ such that $T\left(x_\varepsilon^*\left(c\right);c\right) < T\left(s;c\right)$ for all $c < c_1$.*
2. *There exists an interval of c, (c_2, c_3), with $\bar{c} \in (c_2, c_3)$, such that $T\left(x_\varepsilon^*\left(c\right);c\right) > T\left(s;c\right)$ generically for all $c \in (c_2, c_3)$.*
3. *There exists a threshold $c_4 \geq \bar{c}$ such that $T\left(x_\varepsilon^*\left(c\right);c\right) < T\left(s;c\right)$ for all $c > c_4$.*
4. *If $T(x_\varepsilon^*\left(c\right);c)$ is concave in c, then $c_2 = c_1$ and $c_3 = c_4$.*

This functional-form-free type-by-type characterization reveals a fundamental trade-off. For injurers with costs near the population average (part 2), the simple standard is already set at or near their personal optimum, so that legal uncertainty increases social costs by distorting behavior away from the efficient standard, nesting the classic result of (Craswell and Calfee, 1986). Conversely, for more "extreme" types with sufficiently low or high costs (parts 1 and 3), legal uncertainty is affirmatively welfare-enhancing by breaking inefficient bunching at an ill-fitting standard and allowing differentiated behavior that better reflects individual costs.

The certainty benchmark in Proposition 3 is held fixed at compliance with the standard to isolate the smoothing effect. Very high-cost injurers ($c > \tilde{c}$), however, would opt out under certainty and take first-best care, which uncertainty cannot improve. Corollary 1 accounts for this to complete the welfare analysis.

Corollary 1 (Social costs with opt-out types)**.** *Let $\tilde{T}\left(c\right)$ be the social costs under certainty: $\tilde{T}\left(c\right) = T\left(s;c\right)$ for $c \leq \tilde{c}$ and $\tilde{T}\left(c\right) = T\left(x^{FB}\left(c\right);c\right)$ for $c > \tilde{c}$. Then $T\left(x_\varepsilon^*\left(c\right);c\right) - \tilde{T}\left(c\right)$ follows* *the sign pattern in Proposition 3 for $c \leq \tilde{c}$. For $c > \tilde{c}$, $T\left(x_{\varepsilon}^{*}\left(c\right);c\right) - \tilde{T}\left(c\right) > 0$: uncertainty strictly raises social costs because certainty implements $x^{FB}(c)$ for these types.*

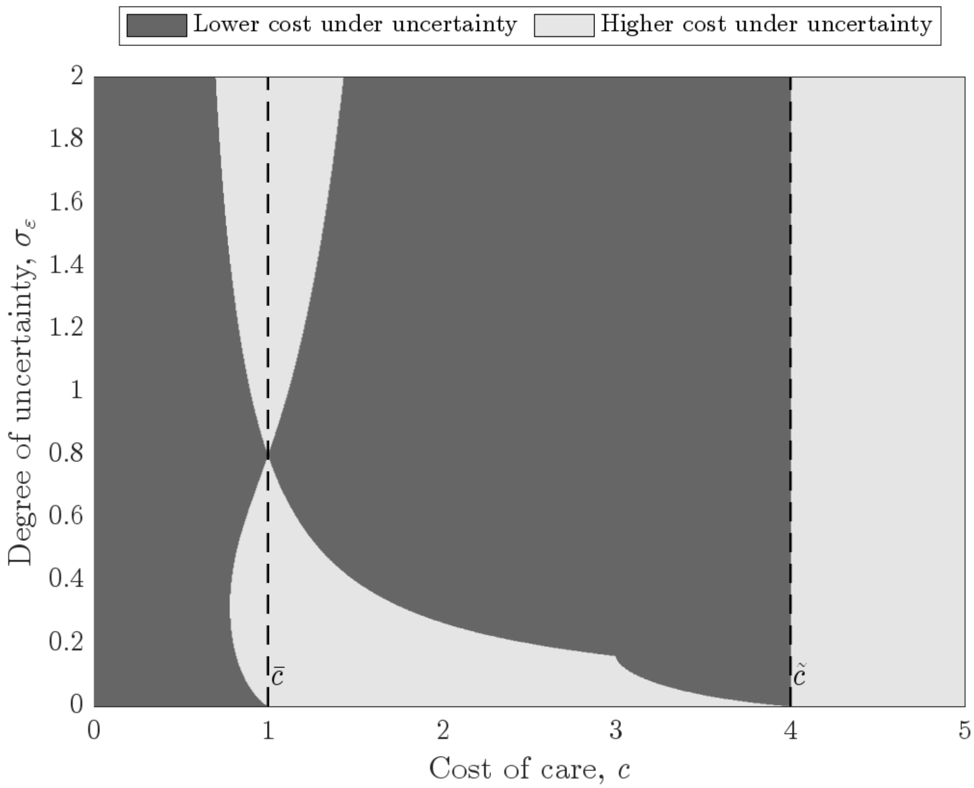


Figure 4: **Sets of types c with lower vs. higher social costs of accidents under uncertainty than under certainty in the legal noise model.** Assumptions: $l\left(x\right) = \frac{1}{x}$ and $\bar{c} = 1$ so that $\tilde{c} = 4$ and $s \equiv x^{FB}\left(\bar{c}\right) = 1$, with injurers' beliefs about s distributed $\mathcal{N}\left(1, \sigma_{\varepsilon}^{2}\right)$ for varying levels of σ_{ε}.

Figure 4 illustrates these results by showing the ranges of types c with lower total accident costs under legal uncertainty than under certainty for varying degrees of uncertainty (σ_{ε}). Beginning from $\sigma_{\varepsilon} = 0$ along the horizontal axis, introducing a small amount of legal uncertainty improves behavior for all injurers with $c < \bar{c}$ by increasing their care, but worsens the behavior for injurers with $c \geq \bar{c}$. At higher levels of uncertainty there always exists an interval of types c around $\bar{c}$ for which uncertainty raises accident costs, as well as a set of more extreme types for which uncertainty lowers accident costs. The dilution effect of very high levels of uncertainty is reflected in the widening range of types for which uncertainty worsens behavior as uncertainty increases in the top half of the figure. The key insight is that legal uncertainty tends to worsen care taken for types near the average cost and for very high cost types, but tends to improve care for types that are substantially above or below the average cost.

The aggregate welfare effect of legal uncertainty follows directly from this type-by-type characterization; it is simply the integral of these competing effects weighted by the population density. The net effect of legal uncertainty on total accident costs across the population thus depends on the weight that $f(c)$ places on the different intervals of types characterized above. To illustrate how these forces aggregate, Figure 5 depicts the region of the parameter space $(\sigma, \sigma_\varepsilon)$ where total social costs are lower under uncertainty than under certainty, assuming $f(c)$ follows a lognormal distribution and using a specific functional form for $l(x) = \frac{1}{x}$. When there is no heterogeneity across injurers ($\sigma = 0$), legal uncertainty always worsens behavior overall, since the mean type takes first-best care under certainty. As heterogeneity increases, the range of degrees of uncertainty that improve welfare expands.

The smoothing channel is potentially operative only when, absent legal uncertainty, there would be discontinuities in marginal legal incentives as behavior changes, as illustrated in the case of the negligence rule in Figure 3. When such discontinuities exist, any form of legal uncertainty that makes expected legal outcomes vary smoothly with behavior will produce more differentiated incentives. This includes substantive legal uncertainty (where individuals do not know the legal thresholds generating the discontinuities) and procedural legal uncertainty (where individuals are uncertain about how the legal system will measure their behavior). Sanctions regimes that delineate legally permissible behavior from legally prohibited behavior are a primary example of discontinuities in marginal legal incentives that implicate the smoothing channel, as our model of the reasonable person standard illustrates. Another example includes legal regimes that use thresholds of behavior to determine the classification of an activity for regulatory or tax purposes, such as whether a financial instrument is taxed as equity or as debt.

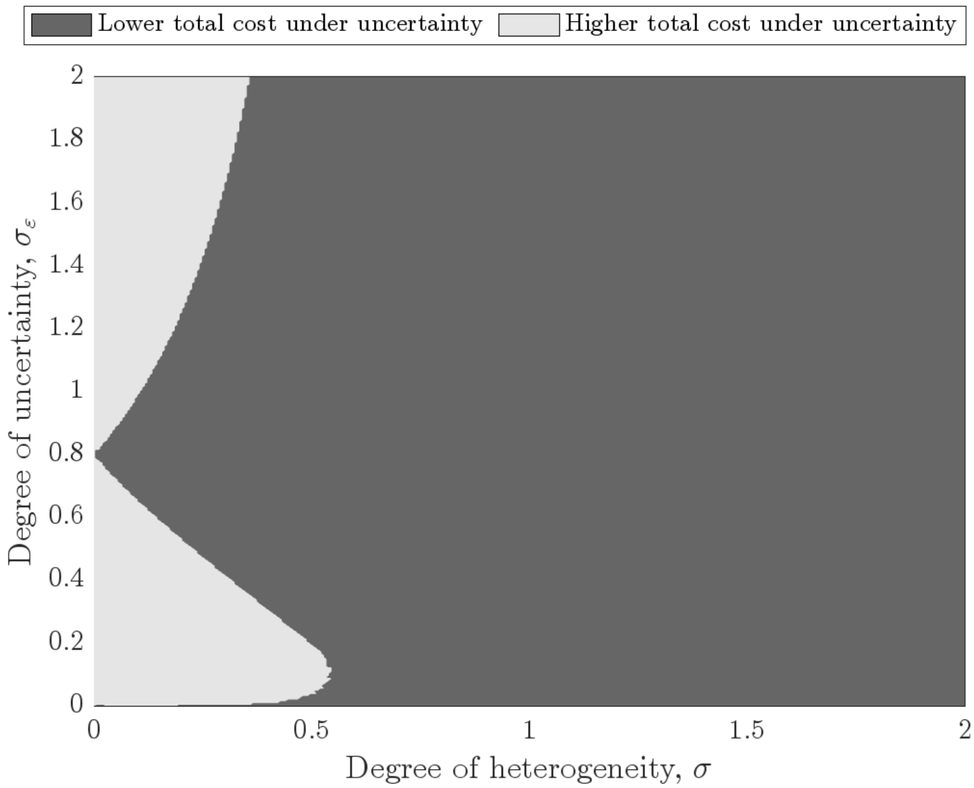


Figure 5: **Parameters with lower vs. higher total social costs of accidents under uncertainty in the legal noise model.** Assumptions: $l(x) = \frac{1}{x}$ and $c \sim \mathcal{LN}(\theta, \sigma^2)$, with $\theta = -\frac{1}{2}\sigma^2$ so that $\bar{c} = 1$, $\tilde{c} = 4$, and $s \equiv x^{FB}(\bar{c}) = 1$, with injurers' beliefs about s distributed $\mathcal{N}(1, \sigma_\varepsilon^2)$ for varying levels of σ_ε.

Importantly, the smoothing mechanism operates by making expected legal outcomes *differentiable*, not by affecting the continuity of legal outcomes *per se*, contra existing accounts in the literature.[9] Consider for example the cause-in-fact requirement of tort law, which might reduce or even eliminate the discontinuity in expected legal outcomes produced by the negligence rule (Grady, 1987; Kahan, 1989). Even if injurers are only liable for accidents that would not have occurred had they taken due care—making their ex-

[9]See, e.g., Bar-Gill and Ben-Shahar (2003, pp. 454 - 458) (arguing that the "self-selection mechanism" identified by Rubinfeld (1987) functions by removing discontinuities in expected legal outcomes and noting that, under the incremental damages model of Grady (1987) and Kahan (1989), "the negligence regime does *not* in fact set a discontinuous payoff structure"); Weisbach (1999, pp. 872 - 873) (analyzing "discontinuous" rules in which "[m]oving one step to the left will cause a large change in tax consequences"); and Fox and Goldin (2019, p. 244) (focusing on "the discontinuous relationship between legal inputs (e.g., taxpayer characteristics) and legal outputs (e.g., tax liability)."). In contrast, Rubinfeld (1987, p. 388)'s analysis of comparative negligence as a smoothing device focused correctly on continuity of *marginal* legal incentives, not on continuity of legal outcomes.

pected liability continuous through $x = s$—there remains a discontinuity in their private *marginal* benefit from care at $x = s$, causing bunching at that point for injurers with $x < \bar{c}$. Consequently, the smoothing mechanism remains potentially operative under legal uncertainty. We provide an analysis of negligence with incremental damages in Appendix B.

Finally, the differentiation in care across injurers under uncertainty moves in the socially desirable direction in this model—i.e., $\frac{\partial x_{\varepsilon}^{*}(c)}{\partial c} < 0$ and $\frac{\partial x^{FB}(c)}{\partial c} < 0$—because the cross-partials with respect to type and care have the same sign in both the injurer's objective function and the social objective function. In fact, in this simple model they also have the same magnitude (1), since the private marginal cost of care equals the social marginal cost of care. These cross-partials typically go the same way because the social objective function is an aggregation of individual payoffs, including those of the regulated actors.

However, externalities that the law fails to internalize may cause the cross-partials to move in opposite directions. This can occur, for example, in a setting in which injurers vary in their degree of dangerousness in such a way that more dangerous injurers face higher opportunity costs of care but also produce greater social benefits from taking greater care. In antitrust contexts, firms may vary in how effectively certain anticompetitive strategies (e.g., exclusive dealing, loyalty rebates) can increase market power—which is privately profitable but socially harmful—and the social costs may not be fully borne by the firm. In these settings, the relevant cross-partials of the private and social objective functions may have opposite signs, potentially resulting in differentiation that moves in a socially undesirable direction.

2.4 Reasonable person standard with updating

We now examine a different form of legal uncertainty that arises when injurers do not know the standard of care but form Bayesian beliefs based on their own information. While the previous model of "legal noise" assumed injurers shared common beliefs about the standard, here we explore how heterogeneous beliefs emerge when injurers use their own characteristics to infer what the law requires.

Specifically, suppose that injurers do not know the standard of care s or the average cost of care in the population $\bar{c}$, but understand they are required to take "reasonable care"—meaning the optimal level of care for an average-cost injurer, $s = x^{FB}(\bar{c})$. Injurers must therefore form beliefs about s based on their beliefs about $\bar{c}$.

To make the analysis tractable, we introduce three functional-form assumptions. First, we assume that $l(x) = \frac{1}{x}$, which will be used to prove the monotonicity of injurers' care choices in Proposition 5 and for our numerical examples. Under this assumption, the first-best level of care that solves the first-order condition in (2) is $x^{FB}(c) = c^{-\frac{1}{2}}$.

Second, we assume that the costs of care in the population follow a lognormal distribution, which restricts injurers' costs of care to be non-negative. That is, we assume that $c = e^{\tau}$ where τ is a normally distributed "personal characteristic"—such as experience, expertise, reaction time, or access to technology—with mean θ and variance σ^2. Injurers' costs c thus have mean $\bar{c} = e^{\theta + \frac{\sigma^2}{2}}$. Since there is a one-to-one relationship between c and τ, we can refer to the injurer's type using either parameter.

Third, we assume that injurers share a common, unbiased prior about θ, distributed $\mathcal{N}(\theta, \sigma_0^2)$. This ensures that injurers' prior and posterior beliefs about θ have the same functional form (i.e., are conjugate distributions).[10] For simplicity we assume that σ^2 and the overall structure of the model (including functional forms) are common knowledge, so the only relevant unknown for the injurers is θ.

Each injurer's information set about $\bar{c}$ consists of their prior and a single draw from the distribution of c, namely their own type. To derive injurers' posterior beliefs about the standard of care, we work first with their beliefs about θ, the underlying mean of

[10] Assuming a normal (instead of lognormal) distribution of the costs of care would also result in conjugate distributions for beliefs and in some ways simplify the analysis, but at the cost of introducing negative costs of care, which would pose a problem—and hence require additional assumptions—when constructing the social objective function. Additionally, allowing priors to be biased—that is, distributed $\mathcal{N}(\mu_0, \sigma_0^2)$ for some μ_0 possibly different from θ—would only add a layer of analysis without affecting our results qualitatively.

$\tau = \log(c)$. This allows us to use standard results on Bayesian updating with normal distributions to derive injurers' beliefs about θ, which in turn pin down their beliefs about $\bar{c} = e^{\theta + \frac{\sigma^2}{2}}$ and thus about $s = x^{FB}(\bar{c}) = e^{-\frac{1}{2}\left(\theta + \frac{\sigma^2}{2}\right)}$.

An injurer of type c has a signal of θ, $\tau = \log(c)$, which is distributed $\mathcal{N}(\theta, \sigma_0^2)$. Their posterior beliefs about θ follow the normal signal updating rule and are distributed normally with mean $\mu_1(c)$ and variance σ_1^2, given by:

$$\mu_1(c) = \frac{\sigma^2}{\sigma_0^2 + \sigma^2}\theta + \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2}\log(c), \tag{8}$$

and

$$\sigma_1^2 = \frac{\sigma_0^2 \sigma^2}{\sigma_0^2 + \sigma^2}. \tag{9}$$

Intuitively, equation (8) shows that injurers with higher costs of care c have higher expectations of θ and therefore of the mean cost of care in the population, $\bar{c} = e^{\theta + \frac{\sigma^2}{2}}$. Consequently, they have lower expectations about the standard, $s = e^{-\frac{1}{2}\left(\theta + \frac{\sigma^2}{2}\right)}$. We call this the "projection channel": each injurer's own cost of care c informs their beliefs about the reasonable-person standard, resulting in variation in legal incentives that correlates with injurers' costs in a socially useful way.

The extent of differentiation in beliefs can be characterized in terms of the variance of the mean of the posterior beliefs about θ, $\mu_1(c)$, across injurers in the population, which is given by:

$$\text{Var}(\mu_1(c)) = \frac{\sigma^2}{\left(\frac{\sigma^2}{\sigma_0^2} + 1\right)^2}. \tag{10}$$

This equation reveals that differentiation in injurers' expectations about θ depends on the information structure. Greater uncertainty about the population's average characteristic under the prior (higher σ_0^2) leads injurers to put more weight on their own signal increasing variance in expectations about θ. The extent of differentiation also depends on the variance in signals (σ^2) through both the information structure (the σ^2 in the denominator of (10)) and the degree of heterogeneity across injurers (the σ^2 in the numerator of (10)). On net it can be shown that the variance of $\mu_1(c)$ across injurers in the population is increasing in σ^2 if and only if $\sigma^2 < \sigma_0^2$.

Let $f_s(\cdot|c)$ and $F_s(\cdot|c)$ denote the PDF and CDF of injurers' posterior beliefs about the standard s conditional on their type c. Our first key result for the projection channel characterizes these posterior beliefs and shows that injurers with lower c have greater beliefs about the standard of care in a first-order stochastic dominance sense.

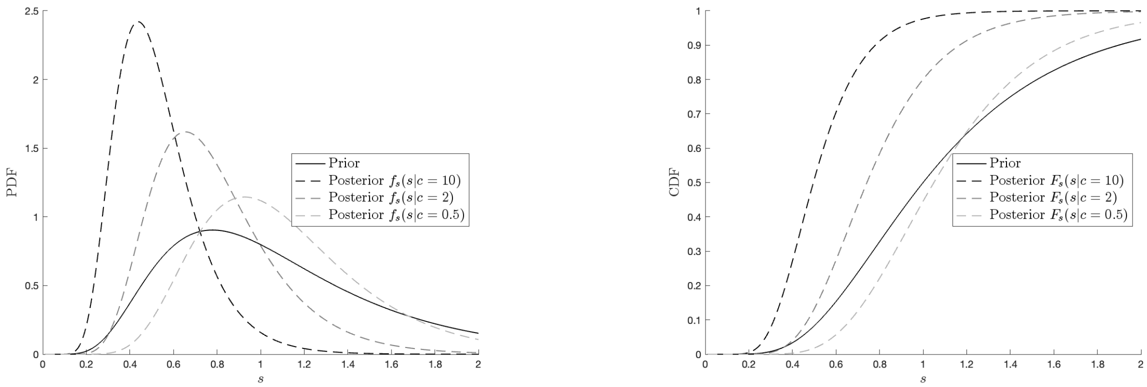
Proposition 4. *Under the reasonable person standard with updating:*

1. *An injurer of type c's posterior beliefs about the standard s are distributed $\mathcal{LN}(\mu_s(c), \sigma_s^2)$ where $\mu_s(c) = -\frac{1}{2}\left(\mu_1(c) + \frac{\sigma^2}{2}\right)$ and $\sigma_s^2 = \frac{\sigma_1^2}{4}$.*

2. *The family of posterior distributions of s parameterized by c satisfies the first-order stochastic dominance property with respect to c: for all $c_1 < c_2$, $F_s(x|c_1) < F_s(x|c_2)$ for all $x \in (0, \infty)$.*

Figure 6 illustrates this result by showing the PDFs and CDFs of injurers' beliefs about the standard s for different draws of c. The differentiation in beliefs is intuitive: injurers with higher costs of care rationally believe that the average cost of care in the population is higher and therefore that the reasonable person standard requires less of them.

This phenomenon resembles empirical findings in psychology where individuals form judgments about the broader population by projecting their own characteristics (Krueger, 2000). For example, survey respondents tend to overestimate how much others share their opinions (Ross et al., 1977), a phenomenon that has been coined the "false consensus effect." While much of the psychology literature attributes such projection to biases like availability bias or motivated reasoning, Dawes (1989) argues that social projection is required by Bayes' rule when one views oneself as a draw from the population distribution, just as we model here.



(a) Posterior PDF of s (b) Posterior CDF of s

Figure 6: **Posterior beliefs about the standard.** Parameters: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$, and $\sigma_0 = 1$.

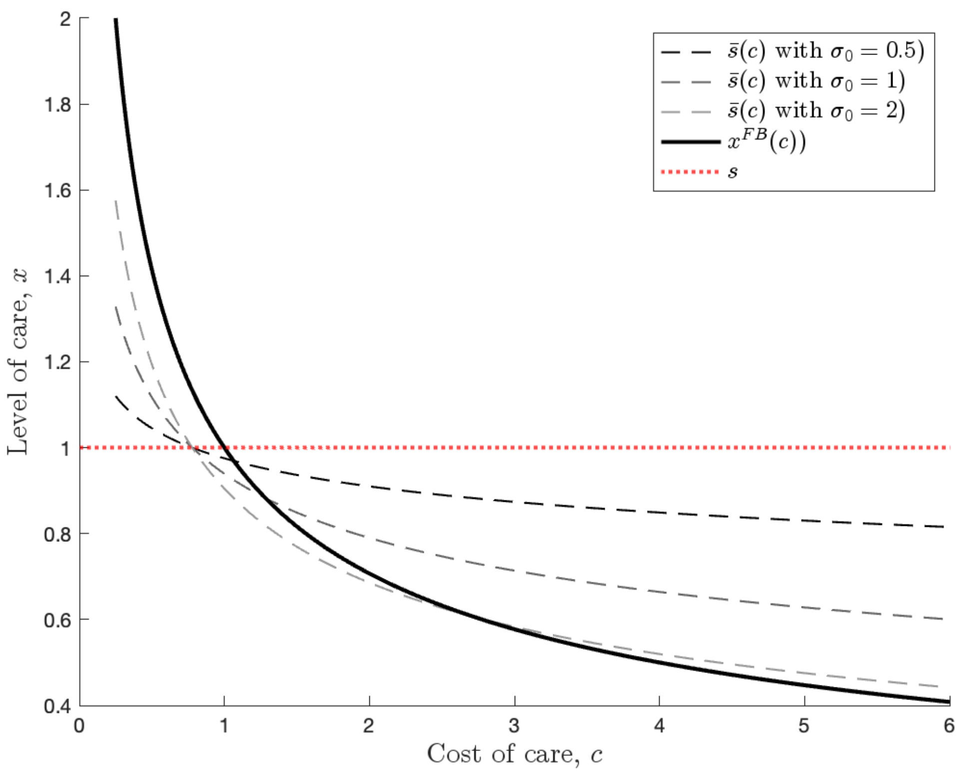


Figure 7: **Injurers' expectations of s with updating.** Assumptions: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ_0.

Figure 7 shows injurers' expected standard of care as a function of c, denoted $\bar{s}(c)$, for different degrees of background legal uncertainty (measured by σ_0), alongside injurers' first-best level of care. With low legal uncertainty (e.g., $\sigma_0 = 0.5$), injurers only slightly update their beliefs based on their type, so $\bar{s}(c)$ is relatively flat. But with high legal uncertainty (e.g., $\sigma_0 = 2$), each injurer updates strongly about $\bar{c}$ based on their own type and expects a standard of care very close to their own first-best level of care, illustrating the projection mechanism at work.[11]

These differences in beliefs about the standard produce differentiated incentives to take care. An injurer of type c chooses the level of care x that solves,

$$\min_{x \geq 0} \Big[\big(1 - F_s(x|c)\big) l(x) + cx \Big]. \tag{11}$$

The injurer's choice of care, $x_s^*(c)$, is implicitly defined by the first-order condition:[12]

$$-\Big[1 - F_s\big(x_s^*(c)\,|c\big)\Big] l'\big(x_s^*(c)\big) + f_s\big(x_s^*(c)\,|c\big) l\big(x_s^*(c)\big) = c. \tag{12}$$

The first-order condition has the same basic form as in the legal noise model (equation (4)), except that the density and distribution functions of injurers' beliefs are now conditional on their type c. Thus, both the projection channel and the smoothing channel operate to produce differentiated incentives.

While it is intuitive that both channels will result in a strictly monotonic negative relationship between injurers' costs of care and levels of care, proving this presents some challenges. In the absence of uncertainty about s, injurers' care is not monotonically increasing with respect to s. At low levels of s, injurers will choose $x = s$ to avoid liability, but when s becomes sufficiently high, they prefer their socially optimal level of care

[11] An observant reader might notice that the $\bar{s}(c)$ functions for different levels of σ_0 all cross at $\bar{s}(0.77881) = s = 1$. This particular signal $c = 0.77881$ leads injurers to not update their unbiased mean prior beliefs and as a result leaves them with an unbiased posterior expectation about the standard, $\bar{s}(c) = s$. With an unbiased posterior expectation, we have $e^{\mu_s(c) + \frac{\sigma_s^2}{2}} = e^{-\frac{1}{2}\left(\theta + \frac{\sigma^2}{2}\right)}$, which—after replacing $\mu_s(c)$ and σ_s^2 and simplifying—yields $c = e^{-\frac{1}{4}\sigma^2}\bar{c}$. It is easy to verify that this implies that the curves in Figure 7 cross at $s = 1$ for $c = e^{-\frac{1}{4}} = .77881$. This signal that results in unbiased posterior expectations is less than $\bar{c}$ because of our lognormal distributional assumption.

[12] This follows from the fact that the solution must be interior. The proof is analogous to the case with noise and is omitted.

$x^{FB}(c) < s$, even though they are then liable for accidents, because the cost of complying with the standard is too great (Shavell, 1987).

More technically, the objective function without uncertainty, $\mathbb{I}_{\{x<s\}} l(x) + cx$, does not satisfy the single-crossing property in s that would ensure monotone comparative statics (Milgrom and Shannon, 1994). This means we cannot rely on standard results from Athey (2002) to guarantee monotone comparative statics in response to FOSD shifts in beliefs about s, nor is the standard implicit function theorem approach tractable. To prove monotonicity, we show instead that the injurers' objective function under uncertainty satisfies the Interval Dominance Order property of Quah and Strulovici (2009).

Proposition 5. *Under the reasonable person standard with updating, the chosen level of care* $x_s^*(c)$ *is strictly monotonically decreasing in the injurer's cost of care,* c*, with* $\lim_{c\to 0} x_s^*(c) = \infty$ *and* $\lim_{c\to\infty} x_s^*(c) = 0$.

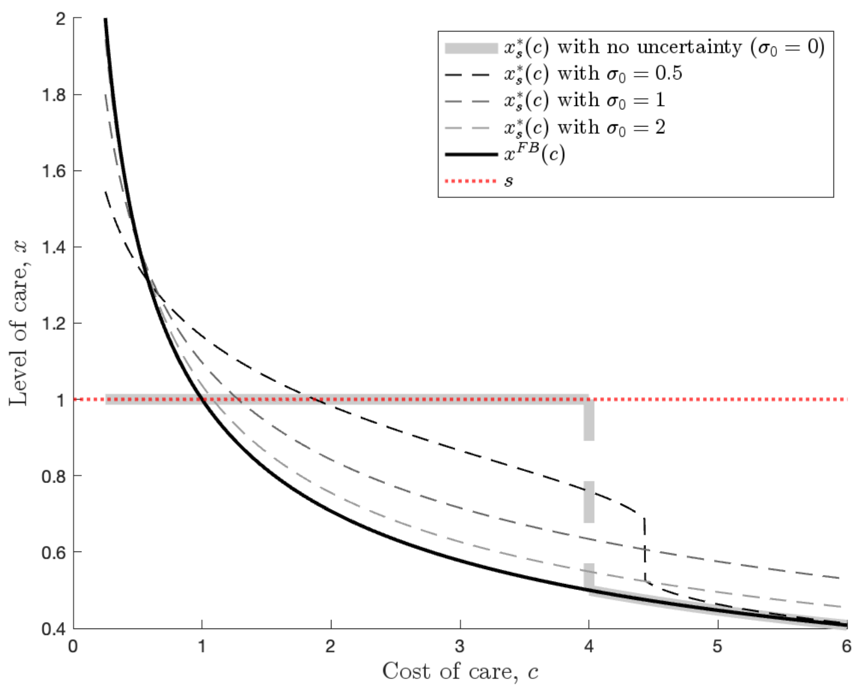


Figure 8: **Care taken with updating.** Assumptions: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ_0.

Figure 8 illustrates the effect of legal uncertainty on injurers' chosen care levels. With low legal uncertainty ($\sigma_0 = 0.5$), care levels are more smoothly differentiated than without uncertainty. Relatively low cost injurers take greater care under uncertainty for two reasons: they expect a standard of care above the true standard of care, due to the projection mechanism, and uncertainty creates incentives to "over-comply" with the expected standard to further reduce the probability of liability. With higher legal uncertainty, the projection mechanism operates more strongly, bringing injurers' care closer to their own first-best level. In the updating model, legal uncertainty always distorts the care choices of injurers with costs of care c in a neighborhood of the average cost of care, $\bar{c}$, and improves care of more extreme types, as in the legal noise model and for the same basic reasons.

Proposition 6. *Under the reasonable person standard with updating, Proposition 3 and Corollary 1 from the legal noise model (characterizing ranges of c types for which legal uncertainty improves or worsens care relative to compliance with a known standard) hold with $x_s^*(c)$ replacing $x_\varepsilon^*(c)$.*

Figure 9 shows the ranges of types with lower accident costs under uncertainty than under certainty for varying degrees of uncertainty.

Figure 10 shows the parameter region where total social costs of accidents is lower under uncertainty than under certainty. As in the legal noise model, the degree of heterogeneity is critical in determining whether uncertainty is beneficial. With low heterogeneity, uncertainty raises the total social cost of accidents by distorting the behavior of types near the mean. With substantial heterogeneity, the social benefits of legal uncertainty kick in. Unlike in the legal noise model, uncertainty remains beneficial even at high levels for the reasons given above.

The projection channel operates only when individuals are uncertain about the substantive content of law and their own type is informative about the relevant legal standards. "Type" here includes not just characteristics of the individuals but also characteristics of the circumstances in which they make decisions that are informative about the general legal standard. The most natural case is when the legal standard is set to be

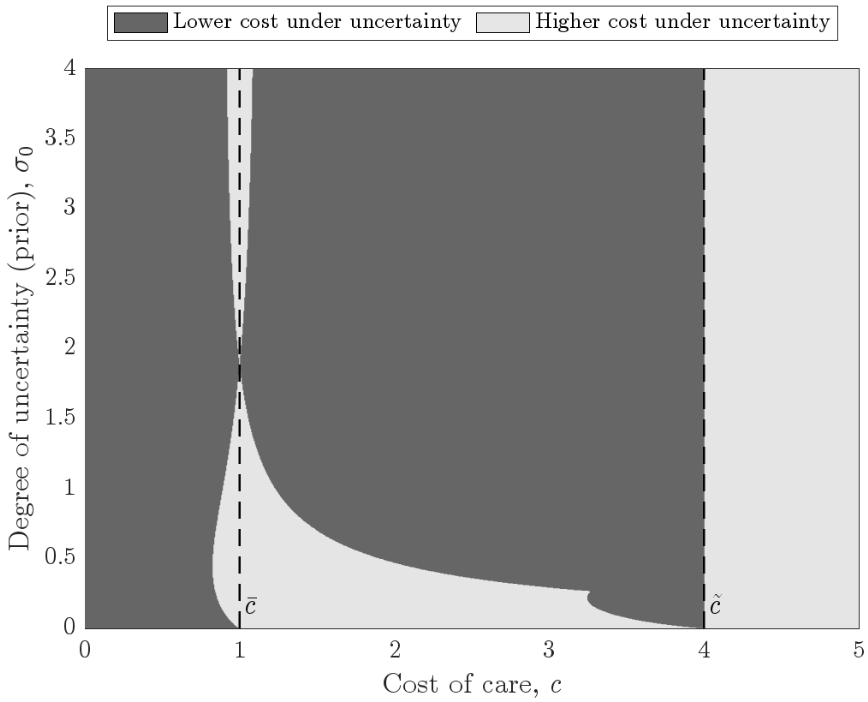


Figure 9: **Sets of types c with lower vs. higher social costs of accidents under uncertainty than under certainty in the updating model.** Assumptions: $l(x) = \frac{1}{x}$, $\sigma^2 = 1.2$, and $\theta = -\frac{1}{2}\sigma^2$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ_0.

appropriate for the "typical" case, but individuals do not know what is typical. This situation is common: individuals often have better information about the costs and benefits of alternative action than the state, while the state often has better information about the broader scope of a problem. In such cases, the optimal legal strategy might be to use a simple vague legal standard, leveraging the projection channel to differentiate incentives to some extent.

While differentiation in beliefs is in a socially useful direction in our model, in other settings the projection channel might operate in a dysfunctional manner. For example, a standard that subjects "abnormally dangerous" activities to sanction might lead injurers to form beliefs about "normal" vs. "abnormal" degrees of dangerousness based on their own activity's dangerousness, resulting in individuals underestimating the likelihood of being sanctioned.

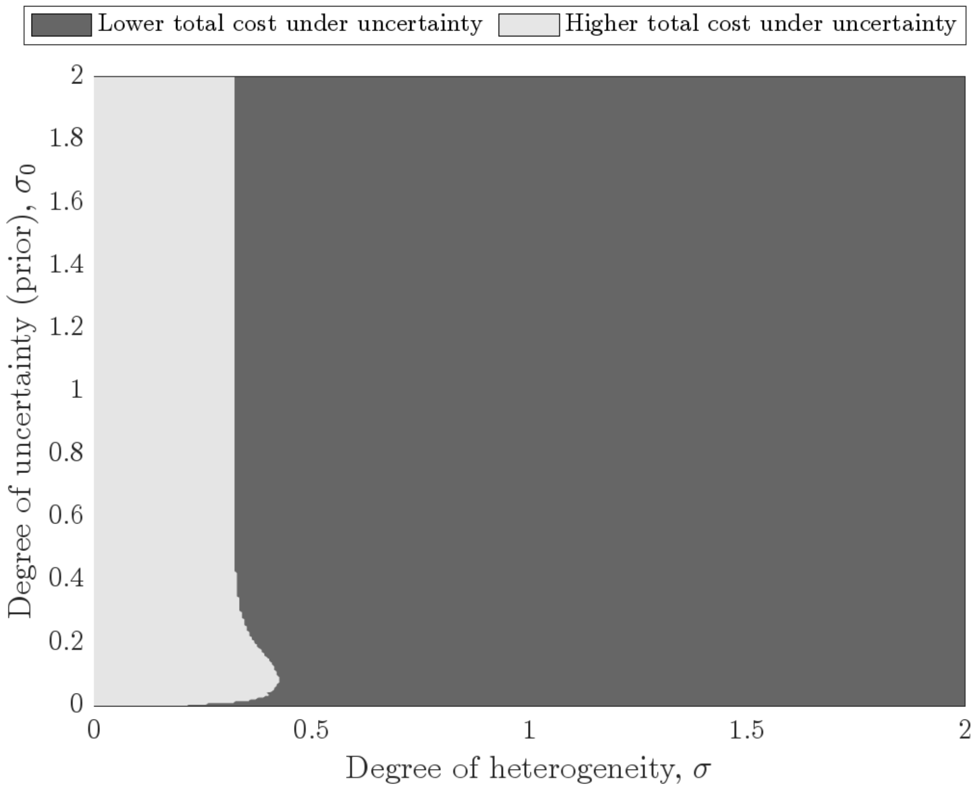


Figure 10: **Parameters with lower vs. higher social costs of accidents under uncertainty in the updating model.** Assumptions: $l(x) = \frac{1}{x}$ and $\theta = -\frac{1}{2}\sigma^2$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ and σ_0.

We have assumed that individuals form beliefs in a rational, Bayesian manner, but deviations from this standard model of belief formation might fundamentally change the projection channel. For instance, "self-serving bias," where individuals conflate what is legal with what benefits themselves (Feldman, 2018), could produce perversely differentiated beliefs about what the law requires.

A more general limitation of the projection channel is that it requires individuals to act on the basis of an incorrect understanding of the law. Experience might correct these misconceptions over time. In contrast, procedural legal uncertainty—and thus the smoothing channel—might be more durable. Still, we think there is substantial room for persistence of substantive legal uncertainty, and therefore the projection mechanism, in the long run. A moment's introspection reveals to us that we ourselves remain ignorant of the content of all but a small fraction of the laws that regulate our behavior in the jurisdictions in which we live and work. Much of human affairs, we submit, is regulated by individuals' best guesses of what the law requires and little else of a legal nature.

In the model with updating, both the smoothing channel and the projection channel operate. Other settings might feature only one of the channels. The legal noise model (Section 2.3) provides an example of a setting in which only the smoothing channel is operative. Conversely, consider the case in which a strict liability rule is used to control the releases of a certain class of hazardous chemicals.[13] Suppose that the chemicals in the class cause varying levels of harm, h, and that the government knows only the average level of harm of the chemicals in the class, $\bar{h}$, not the individual harm levels of each chemical. Individuals are thus held strictly liable for the average harm of chemicals in the class, not their specific harm. Suppose that the individuals subject to this regime know only their own chemical's harm, h, not $\bar{h}$. Here, the projection channel would operate (individuals would form beliefs about the legal sanction using their own harm h as a signal of the average harm $\bar{h}$), but the smoothing channel would not (there is no discontinuity in marginal legal incentives for uncertainty to smooth).

3 Applications to legal design

Our analysis of legal uncertainty and differentiation has significant implications for several fundamental questions in legal design. This section explores how the smoothing and projection channels we identify reshape conventional wisdom on the optimal degree of complexity and personalization of law, the choice between rules and standards, and the choice between prices and sanctions.

[13]We draw inspiration from Kaplow (1992) for this example, albeit we use it to draw quite different conclusions.

3.1 Complexity and personalization

A central challenge in legal design is determining the optimal degree of legal complexity—that is, how finely the law should distinguish among different acts and actors. The conventional approach to this question, developed by Kaplow (1995), suggests that greater heterogeneity in the regulated activity and lower information costs for both the state and individuals call for higher degrees of legal complexity. Building on this framework, Ben-Shahar and Porat (2016) argue that declining information costs in the digital age should lead to more personalized legal standards that differentiate treatment across individuals. Guerra and Hlobil (2018) similarly propose tailoring standards based on observable proxies for injurer type, such as prior accident history.

Our analysis reveals an important qualification to this conventional wisdom. Legal differentiation is not the only way to produce socially useful variation in incentives. Simple legal standards coupled with legal uncertainty can generate differentiated incentives without incurring the information costs that explicit legal differentiation entails. These two differentiation strategies—explicit legal differentiation and differentiation through legal uncertainty—are, to a significant extent, alternatives that the law must choose between rather than deploy simultaneously. As Kaplow (1995) and Kaplow and Shavell (1996) emphasize, explicit legal differentiation requires individuals to predict legal outcomes accurately, which requires low levels of legal uncertainty. Reducing legal uncertainty, however, attenuates the differentiation mechanisms we identify, which operate through uncertainty.

Interestingly, Ben-Shahar and Porat (2016, p. 634) observe that individuals know better what is reasonable for them specifically than what would be reasonable for the "average person." They view this asymmetry as supporting greater personalization of standards of care. Our analysis offers a contrasting perspective: individuals' ignorance of what would be reasonable for the average person, combined with their knowledge of their own characteristics, enables uniform standards to provide differentiated incentives without requiring the state to bear the costs of determining individuals' types. In general then, our analysis suggests that lower levels of legal complexity are optimal than recent work on legal complexity implies.

3.2 Rules versus standards

Our analysis also provides a novel perspective on a classic choice in legal design between rules and standards (Kaplow, 1992). In particular, our results provide a way to reconcile the old idea that standards have a comparative advantage over rules in differentiating behavior (Ehrlich and Posner, 1974; Kennedy, 1975; Schauer, 1991) with the trenchant critique of Kaplow (1992). Kaplow pointed out that this conventional wisdom was based on the implicit assumption that standards are more complex than rules. In principle, however, the complexity of a legal norm can be varied independently of whether it is a rule or a standard, and Kaplow cast doubt on the view that standards are generally more complex than rules. Even standards that ostensibly admit consideration of detailed nuances of the facts may not actually do so in operation. One reason is that, because standards apply case-by-case, it is often optimal ex post for an adjudicator or enforcement body to simplify and consider only the factors most likely to be important, since costly efforts to process information to identify more precisely the optimal legal consequences for the case will have little social benefit when the application of the standard (in its purest form) governs just the instant case.

The relevant choice for legal design, then, is often between *simple* standards, on the one hand, and either complex or simple rules, on the other. In analyzing this choice, Kaplow (1992) assumes both that it is cheaper for individuals to learn about the content of rules than of standards, and that it is socially desirable for individuals to become informed since that results in behavior more in line with legal norms. An implication of these assumptions is that, in settings in which it is highly desirable for the state to differentiate incentives, typically the best way to do so is to deploy a complex rule.

But our analysis of differentiation through legal uncertainty puts the incentive advantages of standards, identified in the prior literature Kaplow was writing against, on firmer microeconomic foundations. In short, standards that in practice operate in a relatively simple, undifferentiated manner can nonetheless produce a usefully differentiated pattern of behavior both by smoothing out discontinuities in legal incentives and by inducing variation in beliefs about the law that correlates with what the law would ideally require. To achieve the same degree of differentiation in behavior using rules, in contrast, would require costly differentiation in legal consequences, since beliefs about rules track more closely the actual content of the rules.

However, the projection channel we identify has an important limitation: it operates through individuals' formation of beliefs about the law based on some announced general standard, like "reasonableness." Rules, in contrast, can convey arbitrary requirements unconstrained by what the beliefs in the population would be about intuitive standards like "reasonableness." The horserace between simple standards and complex rules, then, turns in important part on how effective simple standards would be at differentiating behavior in a particular setting, on the one hand, and the information costs of the state in formulating, and of individuals in navigating, a complex system of rules, on the other.

3.3 Prices versus sanctions

Our analysis also informs the choice in legal design between "sanctions" that impose a detriment for doing what is forbidden and "prices" that specify a payment for doing what is permitted. Cooter (1984) points out that an optimal sanctions regime requires the state to obtain information on optimal behavior, while an optimal pricing regime requires the state to assess external costs accurately. Cooter argues that lawmakers should impose a price to govern an activity if and only if it is cheaper for the state to obtain information about its external costs than to determine optimal behavior; otherwise they should deploy a sanction. Cooter points to heterogeneity in private costs and benefits to the individuals engaged in the activity as one reason it might be cheaper to price behavior than to determine appropriate behavioral norms.

Our analysis suggests that sanctions regimes may be more effective in the face of heterogeneous populations than previously recognized. Both the smoothing and projection channels enable the state to achieve usefully differentiated incentives through a sanction without determining the appropriate behavioral norm for each regulated individual. This broadens the range of circumstances in which sanctions strategies may be viable.

The projection channel can also facilitate price regimes. When individuals know the external cost they impose but not the average cost across individuals engaged in an activity, while the state knows the average external cost but not each individual's external cost, then a vague standard setting price equal to average harm—e.g., injurers will be held "appropriately responsible" for any external costs—can nonetheless differentiate incentives.

4 Conclusion

This paper develops a novel perspective on legal uncertainty, demonstrating how it can serve as a valuable lubricant for the legal system rather than merely a friction to be minimized. We identify two distinct mechanisms through which legal uncertainty enables simple legal standards to provide differentiated incentives without incurring the costs of explicit legal differentiation. First, the smoothing channel eliminates discontinuities in incentives that coarse behavioral standards would otherwise create. This allows injurers with different costs of care to select different levels of precaution, rather than bunching at the legal standard. Second, the projection channel operates through individuals' formation of rational beliefs about what simple standards require, based in part on their own circumstances. This produces differentiation in beliefs about legal standards that correlates with what would ideally be required from each individual. Although we develop these ideas using the reasonable person standard of tort law as a motivating example, this is not a torts paper; these mechanisms operate in material ways, we believe, in many bodies of law.

While we view our analysis as rehabilitating legal uncertainty, from a functional perspective, to a certain extent, we do not claim that legal uncertainty is always socially beneficial once its differentiating effects are recognized. Even in the model we use to develop our analysis, in which differentiation in incentives is socially desirable, we show that legal uncertainty has costs as well as benefits. Rather, we offer a framework for understanding when and how legal uncertainty might enhance welfare by leveraging decentralized information and accommodating heterogeneity at lower administrative cost than explicit legal differentiation.

Finally, our analysis highlights the importance of taking seriously the distinction between the law as written and the law as understood by those subject to it. Much of the economic analysis of law assumes that individuals have perfect knowledge of legal rules and standards. Our work demonstrates that systematic variations in legal beliefs—even those that do not accurately reflect the law—can play a crucial role in shaping behavior and can sometimes enhance rather than detract from law's capacity to induce socially beneficial conduct.

References

Athey, Susan (2002) "Monotone Comparative Statics under Uncertainty," *The Quarterly Journal of Economics*, 117 (1), 187–223.

Baker, Scott and Alex Raskolnikov (2017) "Harmful, Harmless, and Beneficial Uncertainty in Law," *The Journal of Legal Studies*, 46 (2), 281–307.

Bar-Gill, Oren and Omri Ben-Shahar (2003) "The Uneasy Case for Comparative Negligence," *American Law and Economics Review*, 5 (2), 433–469.

Baricz, Árpád (2008) "Mills' Ratio: Monotonicity Patterns and Functional Inequalities," *Journal of Mathematical Analysis and Applications*, 340 (2), 1362–1370.

Ben-Shahar, Omri and Ariel Porat (2016) "Personalizing Negligence Law," *New York University Law Review*, 91 (3), 627–688.

Brown, John Prather (1973) "Toward an Economic Theory of Liability," *The Journal of Legal Studies*, 2 (2), 323–349.

Cooter, Robert (1984) "Prices and Sanctions," *Columbia Law Review*, 84 (6), 1523–1560.

Craswell, Richard and John E. Calfee (1986) "Deterrence and Uncertain Legal Standards," *Journal of Law, Economics & Organization*, 2 (2), 279–314.

Dawes, Robyn M (1989) "Statistical Criteria for Establishing a Truly False Consensus Effect," *Journal of Experimental Social Psychology*, 25 (1), 1–17.

Ederer, Florian, Richard Holden, and Margaret Meyer (2018) "Gaming and Strategic Opacity in Incentive Provision," *The RAND Journal of Economics*, 49 (4), 819–854.

Edlin, Aaron S. and Chris Shannon (1998) "Strict Monotonicity in Comparative Statics," *Journal of Economic Theory*, 81 (1), 201–219.

Ehrlich, Isaac and Richard A. Posner (1974) "An Economic Analysis of Legal Rulemaking," *The Journal of Legal Studies*, 3 (1), 257–286.

Feldman, Yuval (2018) *The law of good people: Challenging states' ability to regulate human behavior*: Cambridge University Press.

Fox, Edward and Jacob Goldin (2019) "Sharp Lines and Sliding Scales in Tax Law," *Tax Law Review*, 73 (2), 237–302.

Grady, Mark F. (1983) "A New Positive Economic Theory of Negligence," *The Yale Law Journal*, 92 (5), 799–829.

——— (1987) "Discontinuities and Information Burdens: A Review of the Economic Structure of Tort Law by William M. Landes Book Review," *George Washington Law Review*, 56 (3), 658–680.

Guerra, Alice and Tobias M Hlobil (2018) "Tailoring negligence standards to accident records," *The Journal of Legal Studies*, 47 (2), 325–348.

Kahan, Marcel (1989) "Causation and Incentives to Take Care under the Negligence Rule," *The Journal of Legal Studies*, 18 (2), 427–447.

Kaplow, Louis (1992) "Rules Versus Standards: An Economic Analysis," *Duke Law Journal*, 42 (3), 557–629.

——— (1995) "A Model of the Optimal Complexity of Legal Rules," *Journal of Law, Economics and Organization*, 11 (1), 150–163.

Kaplow, Louis and Steven Shavell (1996) "Accuracy in the Assessment of Damages," *The Journal of Law and Economics*, 39 (1), 191–210.

Kennedy, Duncan (1975) "Form and Substance in Private Law Adjudication," *Harvard Law Review*, 89 (8), 1685–1778.

Krueger, Joachim (2000) "The Projective Perception of the Social World," in Suls, Jerry and Ladd Wheeler eds. *Handbook of Social Comparison: Theory and Research*, 323–351, Boston, MA: Springer US.

Landes, William M. and Richard A. Posner (1987) *The Economic Structure of Tort Law*: Cambridge, Mass: Harvard University Press.

Lang, Matthias (2017) "Legal Uncertainty as a Welfare Enhancing Screen," *European Economic Review*, 91, 274–289.

Lazarsfeld, PF and RK Merton (1954) "Friendship as a Social Process: A Substantive and Methodological Analysis," in Berger, M ed. *Freedom and Control in Modern Society*, 18–66, New York: Van Nostrand.

Milgrom, Paul and Chris Shannon (1994) "Monotone Comparative Statics," *Econometrica*, 62 (1), 157–180.

Quah, John K.-H. and Bruno Strulovici (2009) "Comparative Statics, Informativeness, and the Interval Dominance Order," *Econometrica*, 77 (6), 1949–1992.

Ross, Lee, David Greene, and Pamela House (1977) "The "False Consensus Effect": An Egocentric Bias in Social Perception and Attribution Processes," *Journal of Experimental Social Psychology*, 13 (3), 279–301.

Rubinfeld, Daniel L. (1987) "The Efficiency of Comparative Negligence," *The Journal of Legal Studies*, 16 (2), 375–394.

Schauer, Frederick (1991) "Rules and the Rule of Law Symposium on Law and Philosophy," *Harvard Journal of Law & Public Policy*, 14 (3), 645–694.

Shavell, Steven (1987) *Economic Analysis of Accident Law*, Cambridge, Mass.: Harvard University Press.

Tabbach, Avraham D. and Alon Cohen (2024) "The Irrelevance of Legal Uncertainty," September, 10.2139/ssrn.4968262.

van Dam, Cees (2014) *European Tort Law*, Oxford: Oxford University Press, 2nd edition.

Weisbach, David A. (1999) "Formalism in the Tax Law," *The University of Chicago Law Review*, 66 (3), 860–886.

Appendix A

A.1 Proof of Proposition 1

If $c \leq \bar{c}$ we have $x^{FB}(c) \geq x^{FB}(\bar{c}) = s$ and hence these injurers minimize their total liability losses by choosing $x = s$. Any greater level of care would only increase their care costs—since they are not liable if $x \geq s$—while any lower level of care would result in greater liability because $l(x) + cx$ decreases in x for $x < x^{FB}(c)$ by convexity of the total cost function. If $c > \bar{c}$, we have $x^{FB}(c) < x^{FB}(\bar{c}) = s$ and hence these injurer make a discrete choice between being liable, $x = x^{FB}(c) < s$—which minimizes their costs among all $x < s$—and abiding by the due-care standard, $x = s$—which minimizes their costs for $x \geq s$. Accordingly, an injurer c takes due care if and only if

$$l\left(x^{FB}(c)\right) + cx^{FB}(c) \geq cs \tag{A.1}$$

and chooses $x^{FB}(c)$, otherwise.

Note that at $c = \bar{c}$, the LHS of A.1 is strictly greater than the RHS. Note as well that the RHS of this inequality increases strictly and linearly in c at a rate of $s = x^{FB}(\bar{c})$ while the LHS is concave in c, increasing at a rate of $x^{FB}(c)$ with second derivative equal to $\frac{dx^{FB}(c)}{dc} < 0$ (where we make use of the envelope theorem, i.e., substitute in using the first-order condition for $x^{FB}(c)$, so that one can ignore any change in $x^{FB}(c)$ when differentiating the LHS with respect to c). Our assumptions on $l(x)$ imply that $\lim_{c\to\infty} x^{FB}(c) = 0$. Therefore, the LHS must cross the RHS from above at a finite level of c denoted $\tilde{c} > \bar{c}$. Injurers with $c \leq \tilde{c}$ face lower total costs if they abide by due care and hence do so, while injurers with $c > \tilde{c}$ violate and choose $x^{FB}(c)$.

Under the specific functional-form assumption we made in constructing the figures in the paper, $l(x) = \frac{1}{x}$, we have that $\tilde{c} = 4\bar{c}$, as can be verified by replacing $l(x) = \frac{1}{x}$ and $s = x^{FB}(\bar{c}) = \bar{c}^{-\frac{1}{2}}$ in (A.1). $\square$

A.2 Proof of Proposition 2

Claim 1.

Strict monotonicity in c. We start by showing that $x^*_\varepsilon(c)$ is strictly monotonically decreasing in c. It is convenient to transform the injurers' minimization problem into a maximization problem using the negation of the injurer's objective function:

$$\max_{x\geq 0} \Pi(x; -c) \quad \text{where} \quad \Pi(x; -c) = -\left(1 - F_\varepsilon(x)\right) l(x) - cx.$$

It is easy to verify that $\lim_{x\to 0} \frac{\partial \Pi}{\partial x} > 0$ and $\lim_{x\to\infty} \frac{\partial \Pi}{\partial x} < 0$ so that for each fixed c there is an interior maximizer. Denote the set of maximizers by

$$X^*_\varepsilon(c) = \arg\max_{x\geq 0} \Pi(x; -c') = \arg\min_{x\geq 0} -\Pi(x; -c'),$$

which, in the generic case, is a single point we label $x_\varepsilon^*(c)$.[14]

Notice that $\Pi(x, -c)$ is continuously differentiable in both arguments, that $\frac{\partial^2 \Pi}{\partial x \partial(-c)} = 1 > 0$—that is, the objective function exhibits increasing marginal returns—and that $x_\varepsilon^*(c) \in (0, \infty) \subset \mathbb{R}$. By standard results in monotone comparative statics (see, e.g., Theorem 1 in Edlin and Shannon (1998)), the set of maximizers $X_\varepsilon^*(c)$ is strictly increasing in $-c$, which it is strictly monotonically decreasing in c. Formally, if $c' > c''$ then $x_\epsilon^*(c') < x_\varepsilon^*(c'')$. This completes the proof that $x_\varepsilon^*(c)$ is strictly monotonically decreasing in c.

Limit of $x_\varepsilon^*(c)$ as $c \to \infty$. We now turn to the the claim $\lim_{c\to\infty} x_\varepsilon^*(c) = 0$. From the first-order condition of the injurer's problem, $x_\varepsilon^*(c)$ must satisfy

$$-[1 - F_\varepsilon(x_\varepsilon^*(c))]l'(x_\varepsilon^*(c)) + l(x_\varepsilon^*(c))f_\varepsilon(x_\varepsilon^*(c)) = c$$

Define:

$$G(x) = -[1 - F_\varepsilon(x)]l'(x) + l(x) f_\varepsilon(x).$$

Then $G(x_\varepsilon^*(c)) = c$. Therefore, we must have $\lim_{c\to\infty} G(x_\varepsilon^*(c)) = \lim_{c\to\infty} c = \infty$. To have $G(x_\varepsilon^*(c)) \to \infty$ as $c \to \infty$, $x_\varepsilon^*(c)$ must head towards an x with $G(x)$ unbounded above.

Observe that $-l'(x)$ is finite and $1 - F_\varepsilon(x) \leq 1$; hence $-l'(x)[1 - F_\varepsilon(x)]$ remains finite as $x \to y > 0$. Likewise, $l(x)$ and $f_\varepsilon(x)$ are finite on $(0, \infty)$; thus the product $l(x) f_\varepsilon(x)$ is also finite on $(0, \infty)$. This shows that $G(x)$ cannot go to infinity for any finite $x > 0$. On the other hand,

$$\lim_{x\to 0} [-l'(x)] = \infty, \quad \text{and} \quad \lim_{x\to 0} [1 - F_\varepsilon(x)] = 1,$$

so $\lim_{x\to 0} G(x) = \infty$. Thus, for large c, the only way $G(x)$ can match x is by having $x \to 0$ so that $G(x) \to \infty$. Hence $\lim_{c\to\infty} x_\varepsilon^*(c) = 0$.

Limit of $x_\varepsilon^*(c)$ as $c \to 0$. We can show that $\lim_{c\to 0} x_\varepsilon^*(c) = \infty$ by using a similar strategy. We have $\lim_{c\to 0} G(x_\varepsilon^*(c)) = \lim_{c\to 0} c = 0$. But $G(x)$ remains strictly positive (and bounded away from zero) for all finite x, while

$$\lim_{x\to\infty} G(x) = 0 \quad \text{(since } l'(x) \to 0 \text{ and } 1 - F_\varepsilon(x),\ f_\varepsilon(x) \to 0 \text{ as } x \to \infty)$$

Thus the only way for $G(x_\varepsilon^*(c))$ to fall to 0 is that $x_\varepsilon^*(c) \to \infty$.

Claim 2. To assess whether an injurer of type c facing a level of uncertainty σ_ε takes more or less care than the standard of care it is sufficient to examine the derivative of the injurer's objective function evaluated at $x = s$. If $x_\varepsilon^*(c)$ is continuous in the relevant

[14] Although strict global convexity need not always hold, any violation of it typically arises on a set of measure zero, so that the (global) minimizer of the cost function is almost always unique. Where non-uniqueness does arise, one can treat $x_\varepsilon^*(c)$ as the set of all global minimizers; the arguments for strict monotonicity below then apply to *any* selection from that set.

range,[15] then $x_\varepsilon^*(c) = s$ only if the derivative above is equal to 0. Then, the level of uncertainty that makes an injurer comply with the standard of care must satisfy:

$$\big(1 - F_\varepsilon(s)\big)\, l'(s) - f_\varepsilon(s) l(s) + c = 0 \tag{A.2}$$

Replacing $F_\varepsilon(s) = \frac{1}{2}$, $f_\varepsilon(s) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon}$, and $l'(s) = l'\big(x^{FB}(\overline{c})\big) = -\overline{c}$, we can write:

$$-\frac{\overline{c}}{2} - \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} l(s) + c = 0$$

or:

$$\hat{\sigma}_\varepsilon(c) \equiv \sqrt{\frac{1}{2\pi}} \frac{l(s)}{c - \frac{\overline{c}}{2}} \tag{A.3}$$

For injurers with $c < \frac{\overline{c}}{2}$, $\hat{\sigma}_\varepsilon(c)$ is not well-defined and it is easy to see that they over-comply for any level of σ_ε. If $\sigma_\varepsilon = \hat{\sigma}_\varepsilon(c)$, then the injurer with cost of care c complies with the standard of care. By strict monotonicity of $x_\varepsilon^*(c)$, all injurers with cost of care $c' < c$ over-comply, $x_\varepsilon^*(c') > s$, and all injurers with cost of care $c'' > c$ under-comply, $x_\varepsilon^*(c'') < s$. Given that $\hat{\sigma}_\varepsilon(c)$ decreases in c, it follows that an injurer of type c over-complies, $x_\varepsilon^*(c) > s$, if $\sigma_\varepsilon < \hat{\sigma}_\varepsilon(c)$ and under-complies, $x_\varepsilon^*(c) < s$, if $\sigma_\varepsilon > \hat{\sigma}_\varepsilon(c)$. □

A.3 Proof of Proposition 3

Define the difference between social costs under uncertainty and under compliance with a known standard of care as:

$$H(c) \equiv l(x_\varepsilon^*(c)) + c x_\varepsilon^*(c) - l(s) - cs = T(x_\varepsilon^*(c); c) - T(s; c)$$

with:

$$H'(c) = (l'(x_\varepsilon^*(c)) + c) \frac{dx_\varepsilon^*}{dc} + x_\varepsilon^*(c) - s \tag{A.4}$$

To simplify, we assume that $x_\varepsilon^*(c)$ is continuous in c.[16]

Step 1. Preliminary analysis and key limits.

We start by establishing key preliminary results and limit behavior.

[15] Recall that, due to the fact that the injurer's objective function is not necessarily globally convex, $x_\varepsilon^*(c)$ is not necessarily continuous although, by Claim 1, it is decreasing in c. Therefore, $x_\varepsilon^*(c)$ and s may cross at a discontinuity point and hence this injurer may never perfectly comply with the standard, but will either over- or under-comply. As a result, the function $\hat{\sigma}_\varepsilon(c)$ defined below may also be discontinuous for some values of c. Considering these case formally would only add details to the proof without affecting our results.

[16] A standard regularity condition that guarantees continuity of $x^*(c)$ is that the second-order condition of the injurer's minimization problem is globally satisfied, which as discussed above is typically the case. If that regularity condition is not satisfied, then there can be measure-zero values of c at which $x^*(c)$ is discontinuous and the derivative $\frac{dx_\varepsilon^*}{dc}$ is undefined. This proof could be extended to handle such measure-zero cases, but doing so would substantially complicate the analysis without adding any insights.

Behavior of $H(\bar{c})$. Recall $s = x^{FB}(\bar{c})$ is the unique global minimizer of $T(\cdot;\bar{c})$. Hence for any $x \neq s$, we have

$$T(x;\,\bar{c}) \;>\; T(s;\,\bar{c}).$$

Consequently, if $x^*_\varepsilon(\bar{c}) \neq s$, then

$$H(\bar{c}) \;=\; T\big(x^*_\varepsilon(\bar{c});\bar{c}\big) \;-\; T\big(s;\bar{c}\big) \;>\; 0.$$

Therefore, whenever $x^*_\varepsilon(\bar{c}) \neq s$, we have $H(\bar{c}) > 0$. If instead $x^*_\varepsilon(\bar{c}) = s$, we have $H(\bar{c}) = 0$.

Limit as $c \to 0$. We claim:

$$\lim_{c\to 0} H\left(c\right) \;<\; 0. \tag{A.5}$$

Recall that $\lim_{c\to 0} x^*_\varepsilon\left(c\right) = \infty$ by Proposition 2. As $x^*_\varepsilon\left(c\right) \to \infty$, we have $l\left(x^*_\varepsilon\left(c\right)\right) = \lim_{x\to\infty} l\left(x\right) = 0$. Also, note that $\lim_{c\to 0}\left[l\left(x^*_\varepsilon\left(c\right)\right) - l\left(s\right)\right] < 0$ and $\lim_{c\to 0} cs = 0$. Hence the crux is to check that $\lim_{c\to 0} cx^*_\varepsilon\left(c\right) = 0$. We isolate this in the lemma below.

Lemma 1. $\lim_{c\to 0} cx^*_\varepsilon\left(c\right) = 0$.

Proof of Lemma 1. Let:

$$G\left(x\right) \equiv -l'\left(x\right)\left[1 - F_\varepsilon\left(x\right)\right] + l\left(x\right) f_\varepsilon\left(x\right)$$

From the first-order condition (4) in the main text, the injurer chooses $x^*_\varepsilon\left(c\right)$ such that $G\big(x^*_\varepsilon\left(c\right)\big) = c$. Notice that $-\,l'\left(x\right) > 0$ and $l\left(x\right) > 0$ for all $x > 0$. Hence for any x,

$$G\left(x\right) = -l'\left(x\right)\left[1 - F_\varepsilon\left(x\right)\right] + l\left(x\right) f_\varepsilon\left(x\right) \leq \alpha\left[\big(1 - F_\varepsilon\left(x\right)\big) + f_\varepsilon\left(x\right)\right]$$

for some $\alpha > 0$ large enough so that $\alpha > -\,l'\left(x\right)$ and $\alpha > l\left(x\right)$. Furthermore, by the normal tail bound (Mills-ratio inequality), for $x > s$:

$$1 - F_\varepsilon\left(x\right) < \frac{f_\varepsilon\left(x\right)\sigma_\varepsilon^2}{x - s}.$$

Thus, one obtains

$$G\left(x\right) < \alpha f_\varepsilon\left(x\right)\left[1 + \frac{\sigma_\varepsilon^2}{x - s}\right] < \alpha f_\varepsilon\left(x\right) = \alpha\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}e^{-\frac{(x-s)^2}{2\sigma_\varepsilon^2}}.$$

Moreover, for $x > 2s$,

$$e^{-\frac{(x-s)^2}{2\sigma_\varepsilon^2}} < e^{-\frac{\left(\frac{x}{2}\right)^2}{2\sigma_\varepsilon^2}}$$

Let $\beta = \frac{\alpha}{\sigma_\varepsilon\sqrt{2\pi}}$, then

$$G\left(x\right) < \beta e^{-\frac{x^2}{8\sigma_\varepsilon^2}}.$$

Because $x_\varepsilon^*(c)$ satisfies $G(x_\varepsilon^*(c)) = c$, we get

$$c < \beta e^{-\frac{\left(x_\varepsilon^*(c)\right)^2}{8\sigma_\varepsilon^2}}.$$

Rearranging and taking logarithms on both sides,

$$-\frac{x_\varepsilon^*(c)^2}{8\sigma_\varepsilon^2} > \log\left(\frac{c}{\beta}\right), \quad \text{i.e.} \quad x_\varepsilon^*(c)^2 < -8\sigma_\varepsilon^2 \log\left(\frac{c}{\beta}\right).$$

Hence

$$0 \le c x_\varepsilon^*(c) < c\sqrt{\log\left(\frac{\beta}{c}\right)}\sqrt{8}\sigma_\varepsilon.$$

As $c \to 0$, the right-hand side goes to 0 since $\lim_{c\to 0} c\sqrt{\log\left(\frac{\beta}{c}\right)} = 0$. Hence, by the strict inequality, we also have $\lim_{c\to 0} c x_\varepsilon^*(c) = 0$. □

Consequently, as $c \to 0$, we have that $l\big(x_\varepsilon^*(c)\big) - l(s)$ remains bounded, and $c\, x_\varepsilon^*(c) \to 0$, so $H(c) \to l(\infty) - l(s) < 0$. Thus, we have $\lim_{c\to 0} H(c) < 0$.

Limit as $c \to \infty$. By Proposition 2, we have $\lim_{c\to\infty} x_\varepsilon^*(c) = 0$. Recall:

$$H(c) = \left[l\left(x_\varepsilon^*(c)\right) - l(s)\right] + c\left[x_\varepsilon^*(c) - s\right]$$

As $c \to \infty$, the first bracketed term remains finite (both $l(\cdot)$ and $l(s)$ are finite). Meanwhile, $x_\varepsilon^*(c) \to 0$ ensures $[x_\varepsilon^*(c) - s] \to -s < 0$, so $\lim_{c\to\infty} c\left[x_\varepsilon^*(c) - s\right] = -\infty$. Hence

$$\lim_{c\to\infty} H(c) = -\infty.$$

Step 2. Existence of intervals where $H(c) > 0$ or $H(c) < 0$.

We are now ready to prove the first three claims in the proposition.

Claim 1. Since $\lim_{c\to 0} H(c) < 0$, there exists $c_1 \le \bar{c}$ such that $H(c) < 0$ for all $c < c_1$.

Claim 2. Because $H(\bar{c}) > 0$ (strictly) whenever $x_\varepsilon^*(\bar{c}) \neq s$, it follows by continuity that there is an interval $[c_2, c_3]$ with $c_2 < \bar{c} < c_3$ such that $H(c) > 0$ for all $c \in (c_2, c_3)$ This proves Claim 2 except for the knife-edge case where $x_\varepsilon^*(\bar{c}) = s$, in which case $H(\bar{c}) = 0$, hence the qualifier "generically" in the statement of the proposition.

Claim 3. Since $\lim_{c\to\infty} H(c) < 0$, there is a $c_4 \ge \bar{c}$ such that $H(c) < 0$ for all $c > c_4$.

Step 3. Concavity in c implies at most two sign changes (Claim 4).

Finally, note that

$$\frac{d^2}{dc^2} T\left(x_\varepsilon^*\left(c\right); c\right) = H''\left(c\right).$$

Hence if $T\left(x_\varepsilon^*\left(c\right); c\right)$ is concave in c, the function $H\left(c\right)$ can cross zero at most twice. Combining with the fact that we already found at least two sign changes (negative near $c = 0$, positive around $\bar{c}$, negative again for large c), it follows that there are exactly two zero crossings if $x_\varepsilon^*(\bar{c}) \neq s$ and only one if $x_\varepsilon^*(\bar{c}) = s$. Thus we must have $c_1 = c_2$ and $c_3 = c_4$ in that scenario; and if $\sigma_\varepsilon = \hat{\sigma}_\varepsilon(\bar{c})$, then indeed $c_2 = c_3 = \bar{c}$, and $c_1 = c_4 = \bar{c}$. This proves Claim 4 and completes the proof of Proposition 3. □

A.4 Proof of Proposition 4

With c lognormally distributed with parameters θ and σ^2, the average cost can be written as $\bar{c} = e^{\frac{\sigma^2}{2}} e^\theta$. Note that, since e^θ is lognormally distributed with posterior parameters $\mu_1\left(c\right)$ and σ_1^2, defined in (8) and (9) in the main text, $\bar{c}$ is the multiple of a lognormal variable and hence, using standard formulas, is itself lognormally distributed with parameters $\mu_1\left(c\right) + \frac{\sigma^2}{2}$ and σ_1^2, where $\mu_1\left(c\right)$ is an increasing function of c. In turn, the standard, $s = x^{FB}\left(\bar{c}\right) = \bar{c}^{-\frac{1}{2}}$, is the power of a lognormal variable and hence is also lognormally distributed with parameters $\mu_s\left(c\right) = -\frac{1}{2}\left(\mu_1\left(c\right) + \frac{\sigma^2}{2}\right)$ and $\sigma_s^2 = \frac{\sigma_1^2}{4}$. This proves the first claim in the proposition.

Next, we have $\frac{\partial \mu_s(c)}{\partial c} = -\frac{1}{2c}\frac{\sigma_0^2}{\sigma_0^2+\sigma^2} < 0$ and hence $F_s\left(s|c\right) = \Phi\left(\frac{\log(s)-\mu_s(c)}{\sigma_s}\right)$ is increasing in c, where $\Phi\left(\cdot\right)$ is the CDF of the standard normal distribution. It follows that an increase in c causes a FOSD shift in the distribution, which proves the second claim in the proposition. □

A.5 Proof of Proposition 5

We want to show that $x_s^*\left(c\right)$ is monotonically decreasing in c. To do so we will use the sufficient condition for monotone comparative statics given in Proposition 6 in Quah and Strulovici (2009). We will adopt their notation too.

Let:

$$B\left(x\right) = -\left(1 - F_s\left(x|c\right)\right) l\left(x\right)$$

be the benefit of care and

$$C\left(x\right) = cx$$

be the cost of care so that we can define the objective function $\Pi\left(x\right) = B\left(x\right) - C\left(x\right)$ for $x \in \left(0, \infty\right)$. The cost and benefit functions are differentiable and $C'\left(x\right) > 0$, as required. Note that the injurer's loss function defined in (11) is equal to $-\Pi\left(x\right)$ and hence $x_s^*\left(c\right)$ maximizes $\Pi\left(x\right)$. To show that $x_s^*\left(c\right)$ is monotonically decreasing in c it is sufficient to show that $\tilde{\Pi}\left(x\right)$ dominates $\Pi\left(x\right)$ according to the interval-dominance order, where $\tilde{\Pi}\left(x\right) = \tilde{B}\left(x\right) - \tilde{C}\left(x\right)$ with

$$\tilde{B}(x) = -(1 - F_s(x|\tilde{c}))\, l(x)$$

and

$$\tilde{C}(x) = \tilde{c}x$$

for $\tilde{c} < c$. According to Proposition 6 in Quah and Strulovici (2009), if $B'(x) > 0$, then to show interval dominance it is sufficient to show that there is a positive and increasing function $\alpha(x)$ such that

$$\frac{\tilde{B}'(x)}{B'(x)} \geq \alpha(x) \geq \frac{\tilde{C}'(x)}{C'(x)}.$$

for all $x \in (0, \infty)$. Since $\frac{\tilde{C}'(x)}{C'(x)} = \frac{\tilde{c}}{c} < 1$ we can set $\alpha(x) \equiv \frac{\tilde{B}'(x)}{B'(x)}$ and show that the following three statements hold true for all $x \in (0, \infty)$:

$$B'(x) > 0, \tag{A.6}$$

$$\frac{d}{dx}\left(\frac{\tilde{B}'(x)}{B'(x)}\right) \geq 0, \tag{A.7}$$

and

$$\frac{\tilde{B}'(x)}{B'(x)} \geq 1. \tag{A.8}$$

Note that $F_s(x|c) = \Phi(z)$ and $f_s(x|c) = \frac{1}{x\sigma_s}\phi(z)$ for $z = \frac{\log(x) - \mu_s(c)}{\sigma_s}$, where ϕ and Φ are the PDF and the CDF of the standard normal distribution. Similarly, let $\tilde{z} = \frac{\log(x) - \mu_s(\tilde{c})}{\sigma_s}$. Using $\frac{dz}{dx} = \frac{1}{x\sigma_s}$ and $l(x) = \frac{1}{x}$, we can write:

$$B'(x) = [1 - \Phi(z)]\frac{1}{x^2} + \phi(z)\frac{1}{x^2\sigma_s} > 0.$$

which verifies the condition in (A.6).

Next, we can rewrite the ratio of the marginal benefits as follows:

$$\frac{\tilde{B}'(x)}{B'(x)} = \frac{1 - \Phi(\tilde{z}) + \phi(\tilde{z})\frac{1}{\sigma_s}}{1 - \Phi(z) + \phi(z)\frac{1}{\sigma_s}}.$$

Note that $\frac{dz}{dx} = \frac{d\tilde{z}}{dx} = \frac{1}{x\sigma_s}$. So, by the quotient rule, the following condition is sufficient for $\frac{d}{dx}\left(\frac{\tilde{B}'(x)}{B'(x)}\right) \geq 0$:

$$\left[-\phi(\tilde{z}) + \frac{1}{\sigma_s}\phi'(\tilde{z})\right]\left[1 - \Phi(z) + \phi(z)\frac{1}{\sigma_s}\right] > \left[-\phi(z) + \frac{1}{\sigma_s}\phi'(z)\right]\left[1 - \Phi(\tilde{z}) + \phi(\tilde{z})\frac{1}{\sigma_s}\right].$$

Now note that $\phi'(z) = -z\phi(z)$, so that the condition becomes:

$$\phi(\tilde{z})\left[1+\frac{\tilde{z}}{\sigma_s}\right]\left[1-\Phi(z)+\phi(z)\frac{1}{\sigma_s}\right] < \phi(z)\left[1+\frac{z}{\sigma_s}\right]\left[1-\Phi(\tilde{z})+\phi(\tilde{z})\frac{1}{\sigma_s}\right].$$

Now we multiply the LHS by $\frac{\phi(z)}{\phi(z)}$ and the RHS by $\frac{\phi(\tilde{z})}{\phi(\tilde{z})}$ and simplify the result to obtain:

$$\left[1+\frac{\tilde{z}}{\sigma_s}\right]\left[\frac{1-\Phi(z)}{\phi(z)}+\frac{1}{\sigma_s}\right] < \left[1+\frac{z}{\sigma_s}\right]\left[\frac{1-\Phi(\tilde{z})}{\phi(\tilde{z})}+\frac{1}{\sigma_s}\right]. \tag{A.9}$$

Since we have $\tilde{z} < z$ for $\tilde{c} < c$, we have $1+\frac{\tilde{z}}{\sigma_s} < 1+\frac{z}{\sigma_s}$. Moreover, $\frac{1-\Phi(\cdot)}{\phi(\cdot)}$ is the Mills' ratio and is known to be strictly decreasing in its argument.[17] Thus, for $z > \tilde{z}$ we must have:

$$\frac{1-\Phi(z)}{\phi(z)}+\frac{1}{\sigma_s} < \frac{1-\Phi(\tilde{z})}{\phi(\tilde{z})}+\frac{1}{\sigma_s}.$$

Combining the latter two observations proves that the inequality in (A.9) holds and hence verifies the condition in (A.7). Finally, note that, since $\lim_{x\to 0}\Phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right) = 0$ and $\lim_{x\to 0}\phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right) = 0$, we have:

$$\lim_{x\to 0}\left(\frac{\tilde{B}'(x)}{B'(x)}\right) = \lim_{x\to 0}\left(\frac{1-\Phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right)+\phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right)\frac{1}{\sigma_s}}{1-\Phi\left(\frac{\log(x)-\mu_s(c)}{\sigma_s}\right)+\phi\left(\frac{\log(x)-\mu_s(c)}{\sigma_s}\right)\frac{1}{\sigma_s}}\right) = 1. \tag{A.10}$$

Therefore, since $\frac{\tilde{B}'(x)}{B'(x)}$ is increasing in x according to (A.7), the limit in (A.10) implies that we must have $\frac{\tilde{B}'(x)}{B'(x)} \geq 1$ for all x, which shows that the condition in (A.8) is verified and completes the proof of the strict monotonicity claim. The proof of the limit claims is omitted because it closely follows that of the analogous limit claims in Proposition 2. □

A.6 Proof of Proposition 6

The proof closely follows the proof of Proposition 3 and so we omit it other than the proof of the counterpart to Lemma 1 for the updating model:

Lemma 2. $\lim_{c\to 0} cx_s^*(c) = 0$.

Proof of Lemma 2. From the first-order condition (12) in the main text, the injurer chooses $x_s^*(c)$ such that $K\left(x_s^*(c)\right) = c$, where

$$K(x) \equiv -l'(x)\left(1-F_s(x|c)\right)+l(x)\,f_s(x|c).$$

Notice that $-\,l'(x)$ and $l(x)$ are strictly positive and decreasing in x for all $x>0$. Hence for any $x>0$ we can pick some $\alpha>0$ large enough so that $\alpha > -\,l'(x)$ and $\alpha > l(x)$. It

[17] See Baricz (2008, pp. 1362-1363).

follows that

$$K(x) = -l'(x)\left[1 - F_s(x|c)\right] + l(x) f_s(x|c) \le \alpha\left[\big(1 - F_s(x|c)\big) + f_s(x|c)\right].$$

Since $F_s(\cdot \mid c)$ is lognormal with parameters $\mu_s(c)$ and σ_s^2, by the normal tail bound (Mills-ratio inequality) we have for $x > s$ and $\log x > \mu_s(c)$ (which is the case because we are considering small c and large x):

$$1 - F_s(x|c) = 1 - \Phi\left(\frac{\log x - \mu_s(c)}{\sigma_s}\right) < \frac{\phi\left(\frac{\log x - \mu_s(c)}{\sigma_s}\right)}{\frac{\log x - \mu_s(c)}{\sigma_s}},$$

where, as above, Φ is the standard normal CDF and ϕ is the standard normal PDF. Also,

$$f_s(x|c) = \frac{1}{\sigma_s}\phi\left(\frac{\log x - \mu_s(c)}{\sigma_s}\right).$$

Combining these, for $\log x > \mu_s(c)$, one obtains:

$$\begin{aligned} K(x) &< \alpha\phi\left(\tfrac{\log x - \mu_s(c)}{\sigma_s}\right)\left[\tfrac{1}{\sigma_s} + \tfrac{\sigma_s}{\log x - \mu_s(c)}\right] \\ &< \alpha\tfrac{1}{\sigma_s}\phi\left(\tfrac{\log x - \mu_s(c)}{\sigma_s}\right) \\ &= \alpha\tfrac{1}{\sigma_s\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\log x - \mu_s(c))^2}{\sigma_s^2}}. \end{aligned}$$

Moreover, for $\log x > 2\mu_s(c)$,

$$e^{-\frac{(\log x - \mu_s(c))^2}{2\sigma_s^2}} < e^{-\frac{\left(\frac{\log x}{2}\right)^2}{2\sigma_s^2}}.$$

Pick a constant $\beta = \alpha\frac{1}{\sigma_s\sqrt{2\pi}}$, then

$$K(x) < \beta e^{-\frac{(\log x)^2}{8\sigma_s^2}}.$$

Because $x_s^*(c)$ satisfies $K\big(x_s^*(c)\big) = c$, we get

$$c < \beta e^{-\frac{\left(\log x_s^*(c)\right)^2}{8\sigma_s^2}}.$$

Rearranging and taking logarithms on both sides,

$$-\frac{(\log x_s^*(c))^2}{8\sigma_s^2} > \log\left(\frac{c}{\beta}\right), \quad \text{i.e.} \quad (\log x_s^*(c))^2 < 8\sigma_s^2\log\left(\frac{\beta}{c}\right).$$

Taking square roots and exponentiating gives

$$x_s^*(c) < e^{\sqrt{8\sigma_s^2\log\left(\frac{\beta}{c}\right)}}.$$

Thus

$$0 \leq cx_s^*(c) < ce^{\sqrt{8\sigma_s^2 \log\left(\frac{\beta}{c}\right)}}.$$

As $c \to 0$, the right-hand side goes to 0 since $\lim_{c\to 0} ce^{\sqrt{8\sigma_s^2 \log\left(\frac{\beta}{c}\right)}} = 0$. Hence, by the strict inequality, we must also have $\lim_{c\to 0} cx_s^*(c) = 0$. □

Appendix B

We show here that our main results go through qualitatively unchanged in the model of negligence with incremental damages introduced by Grady (1983) and formalized by Kahan (1989). In this model, there is no discontinuity in the injurer's cost function under certainty. Damages are equal to the harm that is caused by the injurer's negligence, that is, the harm that actually materializes minus the harm that would have occurred anyway had the injurer been nonnegligent, $l(x) - l(s)$. Yet, injurers' marginal total costs (i.e., expected liability costs plus costs of care) are still discontinuous due to the kink in the injurer's costs at $x = s$. This feature of the model dilutes incentives to take care and leads to systematically lower levels of care compared to the basic setup considered above.

B.1 Incremental damages with no uncertainty

If the standard is known, injurers face the following costs:

$$\min_{x\geq 0} \begin{cases} cx & \text{if} \quad x \geq s \\ l(x) - l(s) + cx & \text{if} \quad x < s \end{cases}$$

which leads to the following proposition.

Proposition B.1. *With incremental damages, under a known standard of care s, injurers choose the following levels of care:*

$$x^K(c) = \begin{cases} s & \textit{if} \quad c \leq \bar{c} \\ x^{FB}(c) & \textit{if} \quad c > \bar{c} \end{cases}$$

Proof. Conditional on being negligent, the injurer minimizes $l(x) - l(s) + cx$, which results in the injurer taking care equal to $x^{FB}(c)$. If $x^{FB}(c) \geq s$—that is, if $c \leq \bar{c}$—the injurer is nonnegligent at the first-best level of care and hence the injurer's costs are minimized by reducing care to $x = s$. If instead $x^{FB}(c) < s$—that is, if $c > \bar{c}$—the injurer chooses between the standard, $x = s$, and the first-best level of care, $x = x^{FB}(c) < s$. The latter yields lower total costs if $cs > l\left(x^{FB}(c)\right) - l(s) + cx^{FB}(c)$, which can be written as $l(s) + cs > l\left(x^{FB}(c)\right) + cx^{FB}(c)$. It is easy to see that the inequality is always satisfied by definition of $x^{FB}(c)$ so that the injurer chooses $x = x^{FB}(c)$ in this case. □

Differently from the basic setup (see Proposition 1), in this model injurers either comply with the standard and are under-deterred or violate it and take first-best care. There is no over-deterrence or over-compliance in this case, and a smaller fraction of the population of injurers bunches at the due-care standard.

B.2 Incremental damages with legal noise

With legal noise injurers minimize:

$$\min_{x \geq 0} \left[\int_x^\infty \left[l\left(x\right) - l\left(s\right) \right] f_\varepsilon\left(s\right) ds + cx \right],$$

which yields the following FOC:

$$-\left(1 - F_\varepsilon\left(x_\varepsilon^K\left(c\right)\right)\right) l'\left(x_\varepsilon^K\left(c\right)\right) = c. \tag{B.1}$$

Proposition B.2. *With incremental damages, under the reasonable person standard with legal noise*

1. *The chosen level of care $x_\varepsilon^K(c)$ is strictly lower than the first-best level of care $x^{FB}(c)$, continuous, and strictly monotonically decreasing in the injurer's cost of care, c.*
2. *Injurers with costs of care below a threshold, $c < \bar{c}\left(1 - F_\varepsilon\left(s\right)\right)$, over-comply with the standard, $x_\varepsilon^K(c) > s$, while injurers with costs of care above the threshold, $c > \bar{c}\left(1 - F_\varepsilon\left(s\right)\right)$, under-comply, $x_\varepsilon^K(c) < s$.*
3. *The threshold $\bar{c}\left(1 - F_\varepsilon\left(s\right)\right)$ is lower than $\hat{c}$ in Proposition 2 and is always below the average cost of care, $\bar{c}$, so that the average injurer under-complies.*

Proof. Note the difference between (B.1) and the corresponding FOC of the basic model in (4). Due to the continuity of the injurer's cost function, the term $f_\varepsilon(x)\, l(x)$—which produces incentives towards over-deterrence in the basic model—is missing from (B.1), which leads to under-deterrence for all injurers (i.e., $x_\varepsilon^K(c) < x^{FB}(c)$). The chosen level of care, $x_\varepsilon^K(c)$, decreases in c because the cross-partial derivative is positive,

$$\frac{dx_\varepsilon^K\left(c\right)}{dc} = -\frac{1}{\left(1 - F_\varepsilon\left(x\right)\right) l''\left(x\right) - f_\varepsilon\left(x\right) l'\left(x\right)} < 0,$$

and the SOC is always satisfied. Comparing care taken with due care, there is under-compliance if

$$\left(1 - F_\varepsilon\left(s\right)\right) l'\left(s\right) + c > 0 \tag{B.2}$$

and over-compliance otherwise. Recall that $s = x^{FB}(\bar{c})$, which implies (from the first-order condition that determines $x^{FB}(c)$) that $\bar{c} = -l'(s)$. Substituting that into (B.2) yields $c > \bar{c}\left(1 - F_\varepsilon\left(s\right)\right)$. Under a known standard the threshold for under-compliance was $\bar{c}$ (see Proposition B.1), which is greater than the threshold $\bar{c}\left(1 - F_\varepsilon\left(s\right)\right)$, so that, under negligence with incremental damages, uncertainty dilutes incentives.

To see that $\bar{c}\left(1 - F_\varepsilon\left(s\right)\right) < \hat{c}$ (which was the threshold in Proposition 2) note that $\hat{c}$ is such that

$$\left(1 - F_\varepsilon\left(s\right)\right) l'\left(s\right) - f_\varepsilon\left(s\right) l\left(s\right) + \hat{c} = 0,$$

which compared with the left-hand side of (B.2) yields the result. □

Comparing Proposition B.2 with Proposition B.1 it is easy to see that (1) the smoothing channel is operative in the incremental damages model with legal noise, resulting in more differentiated incentives for injurers to take care; and (2) uncertainty unambiguously dilutes incentives to take care. Furthermore, comparing Proposition B.2 with the corresponding Proposition 2 in the basic model, we can appreciate that incremental damage result in systematically lower levels of care compared to full damages, which results in under-deterrence (compared to the first-best level of care) but may still yield to under- or over-compliance (compared with the due level of care under the standard).

B.3 Incremental damages with updating

The updating process is as in the basic model and hence Proposition 4 applies unchanged to the model with incremental damages. With updating, injurers minimize:

$$\min_x \left[\int_x^\infty (l(x) - l(s)) f_s(s|c)\, ds + cx \right],$$

which leads to the FOC

$$-\left(1 - F_s\left(x_s^K(c)\,|c\right)\right) l'\left(x_s^K(c)\right) = c$$

and the following proposition, mirroring Proposition 5.

Proposition B.3. *With incremental damages, under the reasonable person standard with updating, injurers' levels of care are differentiated:* $\frac{\partial x_s^K(c)}{\partial c} < 0$.

Proof. The SOC is always satisfied and the cross-partial derivative is positive because of FOSD (See Proposition 4). Therefore, we have a unique and interior solution to the injurer's minimization problem, and monotonicity in c:

$$\frac{dx_s^K(c)}{dc} = -\frac{-\frac{d}{dc} F_s(x|c)\, l'(x) + 1}{(1 - F_s(x|c))\, l''(x) - f_s(x|c)\, l'(x)} < 0.$$

□